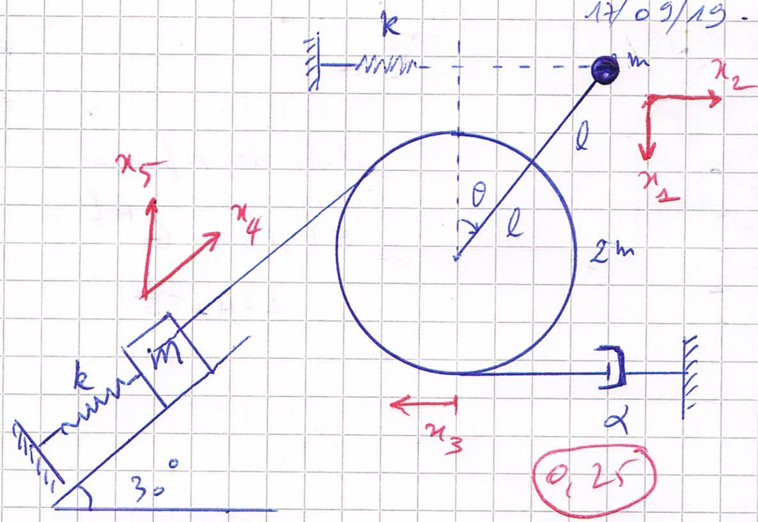
17/09/19.

Exo 2 :



$x_1 = 2l(1-\cos\theta) \simeq l\theta^2$ 0,25

$x_2 = 2l\sin\theta \simeq 2l\theta$ 0,25

$x_3 = l\sin\theta \simeq l\theta$ 0,25

$x_4 = l\theta$ 0,25

$x_5 = x_4 \sin\frac{\pi}{6} = \frac{l}{2}\theta$ 0,25

1) $U = -mg\,x_1 + mg\,x_5 + \frac{k}{2}x_2^2 + \frac{k}{2}(x_4 + x_0)^2$ 1.

$$U = \boxed{-mg\,l\theta^2 + mg\frac{l}{2}\theta + \frac{k}{2}(2l\theta)^2 + \frac{k}{2}(l\theta + x_0)^2}$$

$$U = \left(\frac{5}{2}kl^2 - mgl\right)\theta^2 + \left(mg\frac{l}{2} + klx_0\right)\theta + \frac{k}{2}x_0^2$$ 0,5

2) à l'équilibre : $(\theta = 0)$

$\frac{\partial U}{\partial \theta} = 0 \Rightarrow 2\left(\frac{5}{2}kl^2 - mgl\right)\cancel{\theta} + \left(mg\frac{l}{2} + klx_0\right) = 0 \Rightarrow$

$mg\frac{l}{2} + klx_0 = 0 \Rightarrow \boxed{x_0 = -\frac{mg}{2k}}$ 0,5

3) Pour que l'équilibre soit stable :

$\frac{\partial^2 U}{\partial \theta^2} > 0$ 0,25 $\Rightarrow$ 0,25

$5kl^2 - 2mgl > 0 \Rightarrow 5kl - 2mg > 0$

4) $T = \frac{1}{2}I_1\dot\theta^2 + \frac{1}{2}I_2\dot\theta^2 + \frac{1}{2}m\dot x_4^2$ 1 $\quad (\dot x = l\dot\theta)$

$I_1 = m(2l)^2 \quad ; \quad I_2 = \frac{1}{2}(2m)l^2$

$T = 3ml^2\dot\theta^2$ 0,5

$$D = \frac{1}{2}\alpha\left(\frac{\partial x_3}{\partial \theta}\right)^2 \dot{\theta}^2 = \frac{1}{2}\alpha l^2 \dot{\theta}^2$$ 0,25

$$\mathcal{L} = T - U = 3ml^2\dot{\theta}^2 - \left(\frac{5}{2}kl^2 - mgl\right)\theta^2 - \left(mg\frac{l}{2} + kl x_0\right)\theta + \frac{k}{2}x_0^2$$ 0,5

On a l'équation de Lagrange s'écrit :

$$\boxed{\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) - \frac{\partial \mathcal{L}}{\partial \theta} = -\frac{\partial D}{\partial \dot{\theta}}}$$

1) $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) = 6ml^2\ddot{\theta}$ 0,25

2) $\frac{\partial \mathcal{L}}{\partial \theta} = -(5kl^2 - mgl2)\theta + \cancel{\left(mg\frac{l}{2} + klx_0\right)}$ 0,25

3) $\frac{\partial D}{\partial \dot{\theta}} = \alpha l^2 \dot{\theta}$ 0,25

$$6ml^2\ddot{\theta} + (5kl^2 - mgl2)\theta = -\alpha l^2\dot{\theta} \Rightarrow$$

$$\boxed{\ddot{\theta} + \frac{\alpha}{m6}\dot{\theta} + \left(\frac{5kl^2 - 2mgl}{6ml^2}\right)\theta = 0.}$$ 0,5

$$\lambda = \frac{\alpha}{12m}, \quad \omega_0 = \sqrt{\frac{5kl^2 - 2mgl}{6ml^2}}$$

5) pour avoir un mouvement osaillatoir il faut être dans le régime pseudo-périodique $\Rightarrow \lambda^2 - \omega_0^2 < 0 \Rightarrow$

$$\boxed{\left(\frac{\alpha}{12m}\right)^2 - \left(\frac{5kl^2 - 2mgl}{6ml^2}\right) < 0}$$ 0,5