Corrigé de l'examen final M3 2018/2019

Exercice (1)

1)

① $\int (x+1)^2 dx = \frac{(x+1)^3}{3} + c$

ou bien

$\int (x+1)^2 dx = \int (x^2+2x+1) dx$

$= \frac{x^3}{3} + x^2 + x + c$

② $\int \sin^2 x \, dx = \int \left(\frac{1}{2} - \frac{\cos 2x}{2}\right) dx$

$= \frac{1}{2}x - \frac{\sin 2x}{4} + c$

2)

① $I = \int \frac{1}{x \ln x} dx$

on pose $\ln x = z \Rightarrow dz = \frac{dx}{x}$

Alors:

$I = \int \frac{dz}{z} = \ln|z| + c$

$= \ln|\ln x| + c$

⊛ $I = \int x^2 \ln x \, dx$

$u = \ln x \Rightarrow du = \frac{dx}{x}$

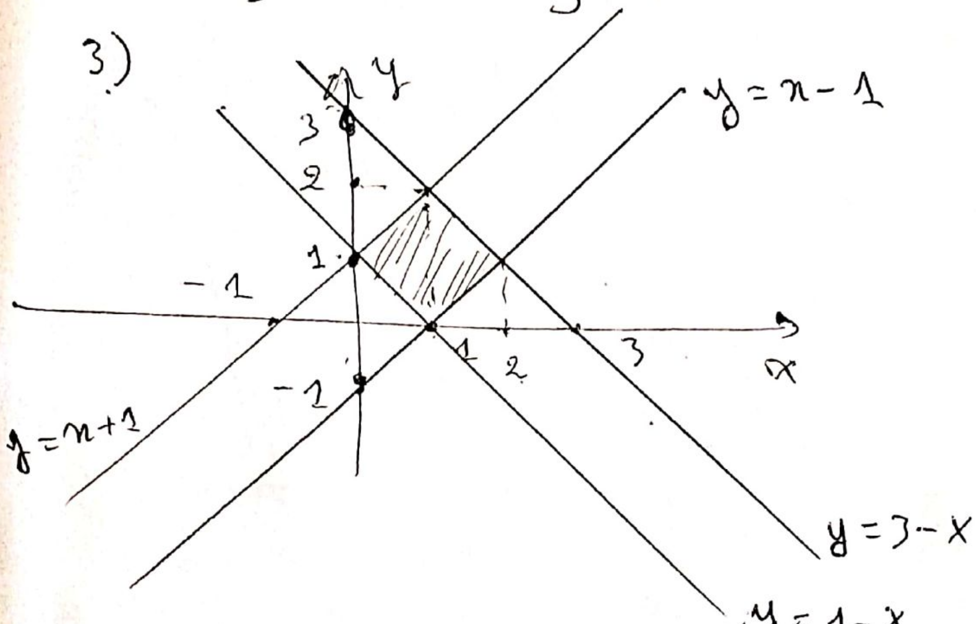
$dv = x^2 dx \Rightarrow v = \frac{x^3}{3}$

Alors,

$I = \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 dx$

$= \frac{x^3}{3} \ln x - \frac{x^3}{9} + c$

3)



$$\boxed{\begin{array}{c} 0 \leq x \leq 1 ; 1 - x \leq y \leq 1 + x \\ \text{et} \\ 1 \leq x \leq 2 ; x - 1 \leq y \leq -x + 3 \end{array}}$$

$$\boxed{\begin{array}{c} 0 \leq y \leq 1 ; 1 - y \leq x \leq y + 1 \\ 1 \leq y \leq 2 ; y - 1 \leq x \leq 3 - y \end{array}}$$

4)

$$I = \int_0^1 \int_2^3 \int_0^x 3x\,dz\,dx\,dy$$

$$= \int_0^1 \int_2^3 3x\,z\Big|_0^x dx\,dy$$

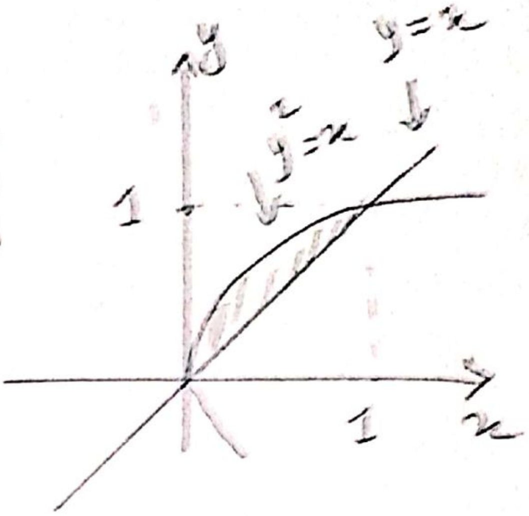
$$= \int_0^1 \int_2^3 3x^2\,dx\,dy$$

$$= \int_0^1 \cancel{3}\frac{x^3}{\cancel{3}}\Big|_2^3 dy$$

$$= \int_0^1 19\,dy = 19y\Big|_0^1 = \boxed{19}$$

Exo 2 :

1. $\int_0^1 \left[\int_0^{\sqrt{x}} f(x,y)\, dy \right] dx$



$\overset{OX}{\Longrightarrow}$

$\Uparrow oy$

Selon oy :

$\begin{cases} 0 \le x \le 1 \\ x \le y \le \sqrt{x} \end{cases}$

Selon ox :

$\begin{cases} 0 \le y \le 1 \\ y^2 \le x \le y \end{cases}$

donc $\int_0^1 \left[\int_x^{\sqrt{x}} f(x,y)\, dy \right] dx \longrightarrow$

$\int_0^1 \left[\int_{y^2}^{y} f(x,y)\, dx \right] dy$

2. $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n} = 2 \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$

série géométrique de raison $\frac{2}{3} \in]-1,1[$

$\left(\left|\frac{2}{3}\right| < 1 \right)$

$S_n = 2 \cdot \frac{2}{3} \cdot \frac{1-(2/3)^n}{1-2/3} = 4\left(1-\left(\frac{2}{3}\right)^n\right)$

$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} 4\left(1 - \left(\frac{2}{3}\right)^n\right) = 4$

donc la série est convergente.

3. $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$

$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e \neq 0 \Rightarrow$ la série est divergente.

Ex ø 3 :

1. $\begin{cases} u = x+y \\ v = x-y \end{cases} \Rightarrow \begin{cases} u+v = 2x \\ u-v = 2y \end{cases} \Rightarrow \begin{cases} x = \frac{u+v}{2} \\ y = \frac{u-v}{2} \end{cases}$

2. $J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} =$

$-\frac{1}{4} - \frac{1}{4} = -\frac{1}{2} \Rightarrow |J| = \frac{1}{2}$

$I = \iint_D \left(\frac{x-y}{x+y}\right)^4 dy\,dx$ devient :

$\iint_{D_1} \frac{v^4}{u^4} \cdot \frac{1}{2} \cdot du\,dv$

Les bornes : D'

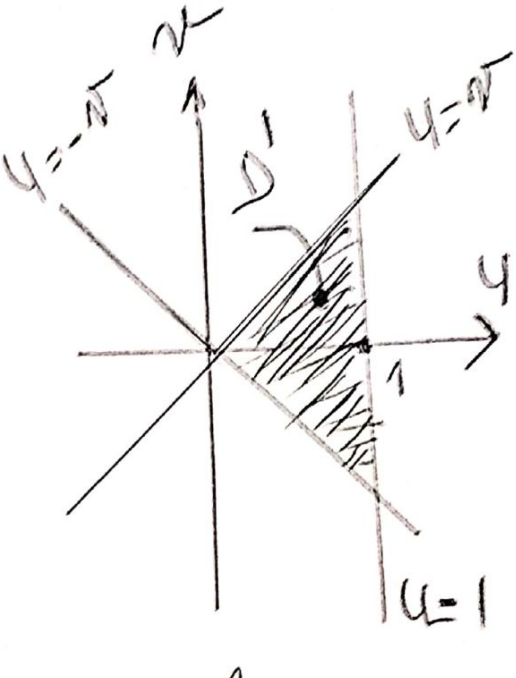
$y = 0 \longrightarrow \frac{u-v}{2} = 0 \Longrightarrow v = u$

$x = 0 \longrightarrow \frac{u+v}{2} = 0 \Longrightarrow u = -v$

$y = 1-x \longrightarrow \frac{u-v}{2} = 1 - \left(\frac{u+v}{2}\right) \Longrightarrow u = 1$

Domaine régulier selon Ov :

$$\begin{cases} 0 \leq u \leq 1 \\ -u \leq v \leq u \end{cases}$$



$$I = \int_0^1 \int_{-u}^{u} \frac{v^4}{u^4} \cdot \frac{1}{2} \cdot dv\, du$$

$$I = \frac{1}{2} \int_0^1 \left[\int_{-u}^{u} \frac{v^4}{u^4}\, dv \right] du$$

$$= \frac{1}{2} \int_0^1 \left[\frac{v^5}{5u^4} \right]_{-u}^{u} du$$

$$= \frac{1}{2} \int_0^1 \left(\left[\frac{u^5}{5u^4} \right] - \left[\frac{-u^5}{5u^4} \right] \right) du$$

$$= \frac{1}{2} \int_0^1 \frac{2}{5} \frac{u^5}{u^4}\, du = \frac{1}{5} \int_0^1 u\, du = \frac{1}{5} \left[\frac{u^2}{2} \right]_0^1$$

donc $I = \frac{1}{5}\left(\frac{1}{2} - 0 \right) = \frac{1}{10}$