ChapitreV : Travail, Puissance & énergie
Chapter V : Power, Work and Energy

1. General concepts
2. Work of a force
3. Power of a force
4. Energy

Power, Work and Energy

- Energy is a fundamental quantity in physics that allows solving certain problems in point mechanics through a scalar equation that could also be solved using the vector form of the fundamental principle of dynamics.
- Energy is the ability to do work. There are various forms of energy in physics, including kinetic energy (energy of motion), potential energy (energy due to position), and many others.
- In summary, work is the energy transfer that occurs when a force acts on an object, energy is the capacity to do work, and power is the rate at which work is done or energy is transferred. These concepts are fundamental in understanding how objects interact in the physical world.

Work of a force

- A force that alters the motion of an object that was initially at rest or causes its deformation does work. Therefore, the work of a force expresses the effort required to move an object. It is denoted as 'W' from the English word work.

The elementary work of the force $\vec{F}$ during the time dt is defined as the dot product of this force with the infinitesimal displacement vector $\overrightarrow{dl}$ also denoted as $d\overrightarrow{OM}$

$$\delta w = \vec{F} \cdot \overrightarrow{dl}$$

δw represents the infinitesimal amount of work done.

We can write:

$$\vec{V} = \frac{\overrightarrow{dOM}}{dt} = \frac{\overrightarrow{dl}}{dt} \qquad \text{donc } \overrightarrow{dl} = \vec{V}\,dt$$

$$\delta w = \vec{F}\,.\,\overrightarrow{dl} = \vec{F}\,.\,\vec{V}\,dt$$

The work done by the force vector $\vec{F}$ along a path AB (or curve C) is equal to the sum of the elementary works. To find the total work over a certain path, you would integrate dW over that path. The integration involves summing up all these infinitesimally small contributions along the entire path.

$$W(\vec{F})_{A\_B} = \sum \delta W = \int_A^B \delta W$$

$$W(\vec{F})_{A\_B} = \int_A^B \vec{F}.\overrightarrow{dl}$$

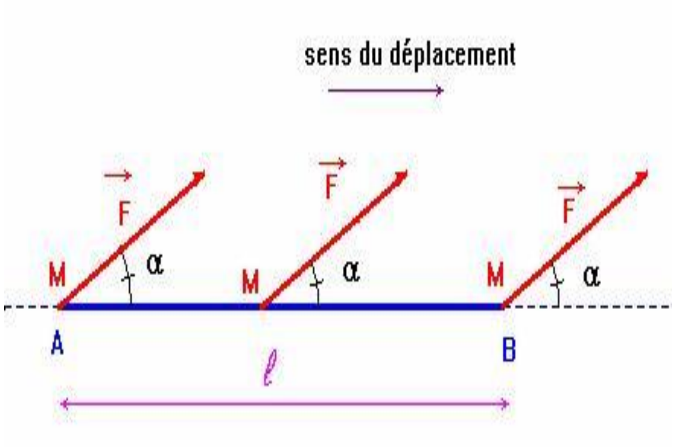
(Travail d'une force constante sur un déplacement rectiligne)

1- Work of a constant force on a rectilinear displacement.

The work of the force on the displacement $\overrightarrow{AB}$ is given by the dot product of this force and the displacement $\overrightarrow{AB}$. The Unit is : N.m , 1N.m = 1 joule

$$\overrightarrow{F} = \overrightarrow{cste} \text{ sur } AB = l \Rightarrow W_{A\rightarrow B}\left(\vec{F}\right) = \int_A^B \vec{F}.\vec{dl}$$

$$\Rightarrow W_{A\rightarrow B}\left(\vec{F}\right) = \vec{F}.\overrightarrow{AB} = F.l.cos\alpha$$

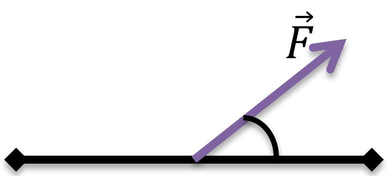


Note : The work is positive, zero or negative, depending on the direction of the force according to the displacement:

Si $\vec{F}$ is perpendicular to $(AB)^{\rightarrow}$ the work is free; The force $F^{\rightarrow}$ does not contribute to the displacement of the object.

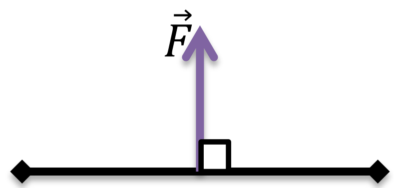
When force opposes displacement, force is resistance and work is negative.

When the driving force of work is positive, (the engine of work, the vehicle that contributes to the movement).



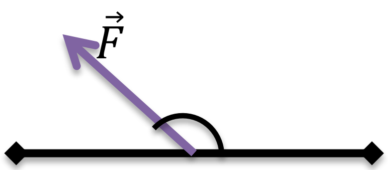
W > O

The work is a driving force



W = O

The work is zero



W ≤< O

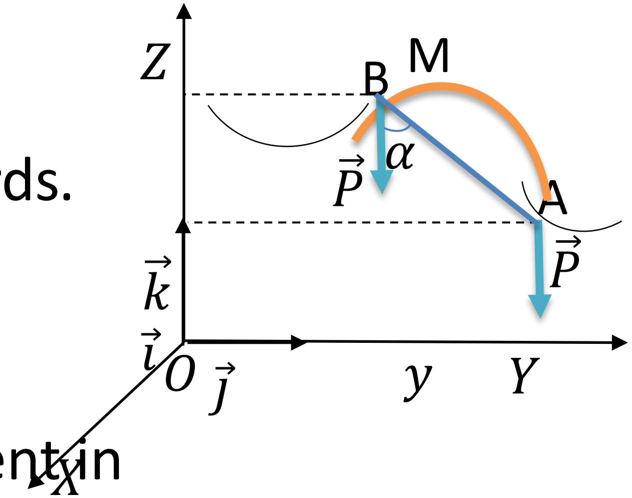
The work is resistant

Travail d'une force constante sur un déplacement quelconque/

Work of a constant force on any displacement

- Let's take, as an example of the work of a constant force, the work done by gravity.
- Consider a material point M with mass m; it is therefore subjected to its weight $\vec{P}$ which is a constant force over time.
- The work done by the weight during a displacement $\overrightarrow{AB}$ is therefore the dot product of the weight $\vec{P}$ = m$\vec{g}$ with the displacement vector $\overrightarrow{AB}$

Example :



Body transported from point A to point B upwards.

Consider the axis reference (O x y z),

$W\left(\vec{P}\right)_{A\to B} = \int_A^B \vec{P}.\overrightarrow{dl} = \vec{P}.\overrightarrow{AB}$ = -P. AB. Cos α

Using the expression for elementary displacement in Cartesian coordinates :$\vec{P} = -mg\vec{k}$ alors :

$$\vec{P}.\overrightarrow{AB} = \left(0\vec{i} + 0\vec{j} - mg\vec{k}\right)\left((x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k}\right)$$

$$\vec{P}.\overrightarrow{AB} = -mg(z_B - z_A)$$

We have $(z_B - z_A) = AB\ cos\ \alpha$ so :

$\vec{P}.\overrightarrow{AB} = -mg(z_B - z_A) = -mg\Delta$h= $-\ mg\ AB\cos\alpha$

In this case the point M therefore rises $(z_B - z_A) > 0$ so $W\left(\vec{P}\right)_{A\to B} < 0$ $(work\ \boldsymbol{resistant})$

We observe that the work done by gravity does not depend on the path taken but only on the difference in altitude between the starting point A and the ending point B.

This property leads to the classification of weight as a conservative force.

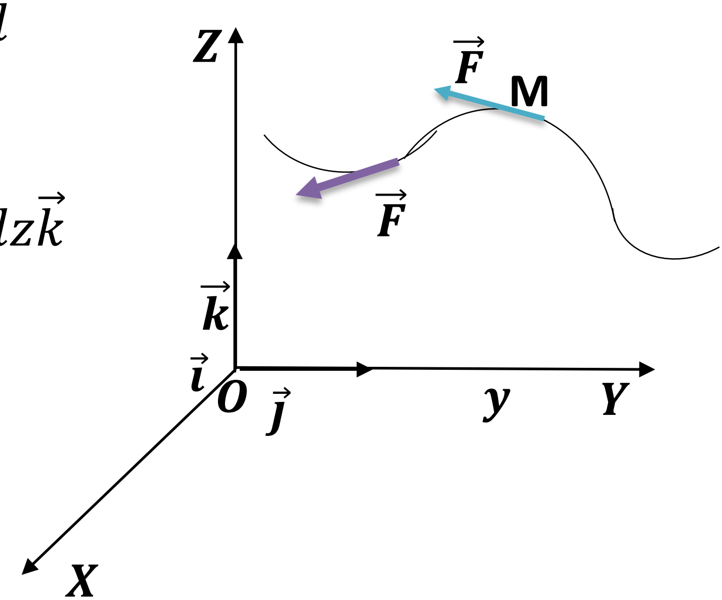
If point M ascends (Δh>0), the work is negative, and it is termed resistant.

If point M descends (Δh<0), the work is positine, and it is termed motive.

The work of a variable force on an arbitrary displacement

- In the case where the force varies in intensity and/or direction during any displacement, integral calculation must be used.

$$A \xrightarrow{W(\vec{F})} B = \int_A^B \vec{F}(M).\overrightarrow{dl}$$



- Using Cartesian coordinates: either:

$\vec{F}(M) = F_x\vec{i} + F_y\vec{j} + F_z\,\vec{k}$ et $d\vec{l} = dx\vec{i} + dy\vec{j} + dz\vec{k}$

- So: $\vec{F}.d\vec{l} = F_x dx + F_y dy + F_z dz$

- we get :

- $A \xrightarrow{W(\vec{F})} B = \int_A^B \vec{F}.\overrightarrow{dl} = = \int_{x_A}^{x_B} F_x.dx + \int_{y_A}^{y_B} F_y.dy + \int_{z_A}^{z_B} F_z.dz$

Let's take the example of a variable force, the elastic force T , or the tension in a spring that varies with the extension state x : $\vec{T} = -Kx\vec{i}$, where K is the spring constant, and x is the displacement .

The work done by the spring tension from position A to position B is given by:

$$W\left(\vec{T}\right)_{A\to B} = \int_A^B \vec{T}.\overrightarrow{dl} = \int_A^B \vec{T}.dx\,\vec{i} = \int_{x_A}^{x_B}(-Kx).dx$$

$$W\left(\vec{T}\right)_{A\to B} = \frac{1}{2}Kx_A^2 - \frac{1}{2}Kx_B^2$$

We also notice that the work of the spring tension depends only on the initial position A and the final position B, therefore the spring tension is a conservative force

Conservative forces القوى المحفوظة

- Conservative forces are those for which the work done by the force in moving an object from one point to another is independent of the path taken. In other words, the total work done by a conservative force is determined only by the initial and final positions of the object and is not affected by the specific path the object follows.

Example : The gravitational force. - The elastic force.

- Work of a constant force in magnitude and direction.

The work done by these forces can be expressed as a change in potential energy

Non-conservative forces القوى غير محفوظة

Non-conservative forces are forces for which the work done in moving an object from one point to another depends on the specific path taken. The total work done by a non-conservative force is not solely determined by the initial and final positions of the object but also by the particular trajectory the object follows.

- Examples of non-conservative forces include friction, air resistance, and some variable forces.

These forces dissipate mechanical energy and typically convert it into other forms such as heat or sound.

Example : The force $\vec{F} = (x^2 - y^2)\vec{\imath} + 3xy\vec{\jmath}$ can go from point A (0, 0) to point B(2,4) following each of the two paths: y = 2x and y = x^2 .
Is this force conservative?

Answer: Following the first path **y = 2x**:
y = 2x → $\vec{F} = (-3x^2)\vec{\imath} + 6x^2\vec{\jmath}$
et dy = 2dx → $\overrightarrow{dl} = dx\vec{\imath} + dy\vec{\jmath} \rightarrow \overrightarrow{dl} = dx\vec{\imath} + 2dx\vec{\jmath}$

$$W\left(\vec{F}\right)_{A\rightarrow B} = \int_A^B \vec{F}.\overrightarrow{dl} = \int_A^B (-3x^2\vec{\imath} + 6x^2\vec{\jmath}).(dx\,\vec{\imath} + 2dx\vec{\jmath})$$

$$= \int_0^2 (-3x^2)dx + 12x^2dx$$

$$W\left(\vec{F}\right)_{A\rightarrow B} = \int_0^2 9x^2dx = 3x^3\Big]_0^2 = 24\,J$$

Following the second path **y =** $\boldsymbol{x^2}$:

y = $x^2 \rightarrow \vec{F} = (x^2 - x^4)\vec{i} + 3x^3\vec{j}$

And dy = 2xdx $\rightarrow \overrightarrow{dl} = dx\vec{i} + dy\vec{j} \rightarrow \overrightarrow{dl} = dx\vec{i} + 2dx\vec{j}$

$$W\left(\vec{F}\right)_{A\rightarrow B} = \int_A^B \vec{F}.\overrightarrow{dl} = \int_A^B ((x^2 - x^4)\vec{i} + 3x^3\vec{j}).(dx\,\vec{i} + 2xdx\vec{j})$$

$$= \int_0^2 (x^2 - x^4)dx + 6x^4dx$$

$$W\left(\vec{F}\right)_{A\rightarrow B} = \int_0^2 (x^2 + 5x^4)dx = \left(\frac{x^3}{3} + x^5\right)\Bigg]_0^2 = 34.6\,J$$

The two works are not equal, so the force in this case is not conservative.

The power of a force : الاستطاعة

The power of a force is the rate at which work is done or energy is transferred by that force. Mathematically, power (P) is defined as the work done (W) per unit of time (t), and it is given by the formula: Pm = $W/\Delta t$ The unit of power is the watt, which corresponds to 1 joule of work done in 1 second, Instantaneous power corresponds to the work done by the force during the infinitesimal time interval dt **:**

$$P(t) = \delta W/dt$$

On a : $W_A^B(\vec{F}) = \int_A^B \vec{F}.\overrightarrow{dl} \rightarrow \delta W = \vec{F}.\overrightarrow{dl}$

Donc : $P(\mathrm{t}) = \frac{\vec{F}.\overrightarrow{dl}}{dt}$ Et $\vec{v} = \frac{\overrightarrow{dl}}{dt}$ $\rightarrow$ $\mathrm{P(t)} = \vec{F}.\vec{v}$ [Watt],

1W=1J/s = 1N.m/s

- A relationship can be established between the work and the power of a force:

$$W_A^B\left(\vec{F}\right) = \int_{tA}^{tB} \vec{F}.\vec{v}.dt = \int_{tA}^{tB} P(t)\,dt$$

Energy :الطاقة

Energy is a fundamental concept in physics that describes the ability of a system to perform work. Energy is a scalar quantity that enables the resolution of numerous problems in dynamics. There are various forms of energy

- Kinetic Energy : **الطاقة الحركية** This is the energy an object possesses due to its motion, so it depends of the velocity of this object : $E_C = \frac{1}{2}mv^2$

The kinetic energy theorem is given by :

$$\sum W_{A\to B}\left(\vec{F}_{ext}\right) = \Delta \boldsymbol{E_C}$$

$$= \boldsymbol{E_C(B) - E_C(A)}$$

$$= \boldsymbol{\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2}$$

The change in kinetic energy of a particle, subjected to a set of external forces between positions A and B, is equal to the sum of the works done by all these forces (both conservative and non-conservative).

Demonstration :

Let us calculate the work of the resultant forces $\vec{F}$ applied to a material point of mass m between two points A and B.

$$W_A^B = \int_A^B \vec{F}.\overrightarrow{dl}$$

and according to the PFD we have : $\vec{F} = m\vec{a} = m\,\frac{d\vec{v}}{dt}$

Hence : $W_A^B = \int_A^B m\,\frac{d\vec{v}}{dt}.\overrightarrow{dl} = \int_A^B m(d\vec{v}.\vec{v})$

With : $\frac{d\vec{l}}{dt} = \overrightarrow{v}$

$$W_A^B = \int_A^B m(v.dv) = \left[\frac{1}{2}mv^2\right]_A^B = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

Potential Energy الطاقة الكامنة

- This is the energy an object possesses due to its position .
- The work done by a conservative force does not depend on the path taken but only on the initial (A) and final (B) states. This work can be expressed using a potential energy function, denoted as Ep.
- The potential energy theorem is given by:

$$\sum W\left(\vec{F}_C\right)_{A\to B} = E_p(A) - E_p(B) = -\Delta E_p$$

- The change in potential energy between two points A and B is equal to the negative of the work done by conservative forces between these two points.

9. Energie mécanique :الطاقة الميكانيكية

The mechanical energy of a system is equal to the sum of the kinetic and potential energies of that system.

$$E_{mec} = E_c + E_p$$

- The theorem of mechanical energy is given by:

$$\Delta E_{mec} = E_{mec}(B) - E_{mec}(A) = \sum W_{A\to B}\left(\vec{F}_{NC}\right)$$

- The change in mechanical energy between two points A and B is equal to the sum of the works done by non-conservative forces between these two points.

Demonstration : The kinetic energy theorem gives :

$$\Delta E_c = E_c(B) - E_c(A) = \sum W_{A\to B}\left(\vec{F}_{ext}\right)$$

and $\sum W_{A\to B}\left(\vec{F}_{ext}\right) = \sum W_{A\to B}\left(\vec{F}_C\right) + \sum W_{A\to B}\left(\vec{F}_{NC}\right)$

$$\sum W\left(\vec{F}_C\right)_{A\to B} = E_p(A) - E_p(B) = -\Delta E_p$$

Donc :

$$\Delta E_c = E_c(B) - E_c(A) = E_p(A) - E_p(B) + \sum W_{A\to B}\left(\vec{F}_{NC}\right)$$

$$\sum W_{A\to B}\left(\vec{F}_{NC}\right) = E_c(B) - E_c(A) + E_p(B) - E_p(A)$$

$$\sum W_{A\to B}\left(\vec{F}_{NC}\right) = \Delta E_c + \Delta E_p = \Delta E_{mec}$$

Principle of conservation of mechanical energy: If the system is conservative or it is only subject to conservative forces or mechanically isolated, then the mechanical energy is conserved.

$$E_c + E_p = E_{mec} = constante$$

This means that the variation in mechanical energy is zero $\Delta E_{mec} = 0$.

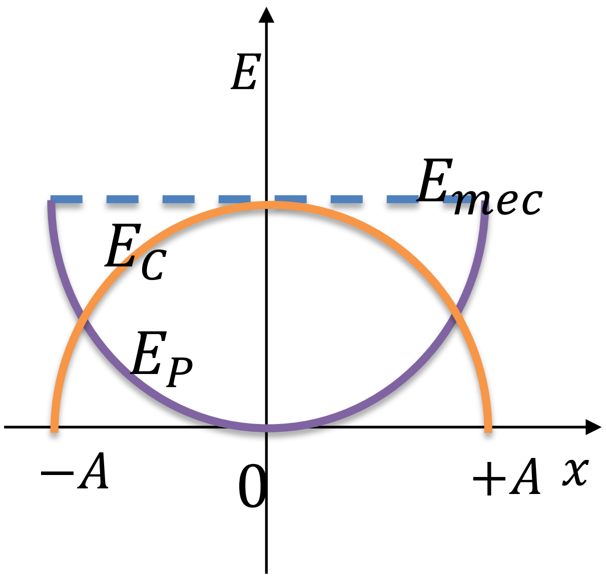
$$E_{mec}(B) = E_{mec}(A)$$

So : $E_C(B) + E_P(B) = E_C(A) + E_p(A)$

Example of conservative systems:

- **System of a mass attached to a spring**

$$E_p = \frac{1}{2}mx^2 \text{ et } \quad E_C = \frac{1}{2}mv^2$$



Simple pendulum

$$E_C = \frac{1}{2}mv^2, \text{ et } \quad E_p = mgh$$

$$E_c + E_p = E_{mec} = const \rightarrow \quad E_{mec}(B) = E_{mec}(A)$$

If the system is not subject to friction, mechanical energy is conserved over time.