جامعـــــة باتنـــــة 2
الشهيد مصطفى بن بولعيد

UE F112
Physics 1

Mécanique du point materiel
Mechanics of Particle

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Chapter I : Mathematical Reviews

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1. Mouvement rectiligne	**Rectilinear motion**
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1/b. Mouvement rectiligne uniformément varié (MRUV)	Uniformly varied rectilinear motion (UVRM)
2. Mouvement circulaire	**Circular movement**
2/a. Mouvement circulaire uniforme	Uniform circular motion
2/b. Mouvement circulaire uniformément varié	Uniformly varied circular motion
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Chapter IV : Dynamics of particles

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2. Systèmes étudiés et actions mécaniques	**Systems and mechanical actions**
3. Différents types de forces	**Types of forces**
4. Lois de Newton	**Newton's Laws**
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Chapitre IV : Dynamique
Chapter IV : Dynamics

5. Application (le pendule simple)	Applications (simple pendulum)
6. Moment d'une force	Moment of a force
7. Moment cinétique	Angular (Kinetic)Momentum
8. Théorème du moment cinétique (TMC)	Kinetic Momentum Theoreme (KMT)
9. Analogie entre grandeurs de translation et de rotation	Analogy between translation and rotation quantities

Chapitre V : Travail, Puissance & énergie
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1. Généralités	General concepts
2. Travail d'une force	Work of a force
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4. Energie	Energy

Chapitre I : Rappels mathématiques
Chapter I : Mathematical Reviews

1. Calcul vectorial — Vector calculus
2. Produit Scalaire — Dot Product (Scalar product)
3. Produit vectoriel — Cross Product (Vector Product)
4. Systèmes usuels de coordonnés — Usual coordinate systems

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1. Calcul vectoriel Vector Calculus

Introduction to Vectors

Before we dive into vectors, it's important to understand the distinction between scalars and vectors:

- **Scalars** are quantities that have magnitude (size) only, such as distance or speed.
- **Vectors** are quantities that have both magnitude and direction, such as displacement, velocity or force.
- **1. Grandeur scalaire : Scalar quantity**

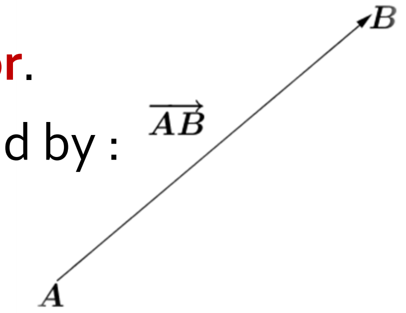
A **scalar** quantity is expressed by a numerical value followed by the corresponding unit.

Exemple: the length, mass, volume , temperature, time, work, voltage, density, resistance,etc ….

2. Grandeur vectorielle : Vector quantity

A quantity that has **magnitude** as well as **direction** is called a **vector**.

A directed line segment AB, having an origin A and an end B, defined by :



- ✓ Its origin
- ✓ Its direction
- ✓ Its magnitude (length)

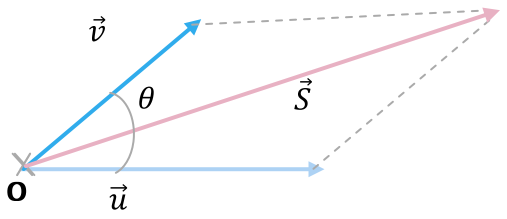
Vectors are often represented graphically as arrows. The length of the arrow represents the vector's magnitude, and the direction of the arrow indicates its direction.

like displacement, velocity, acceleration, force, weight, momentum, electric field intensity etc…..

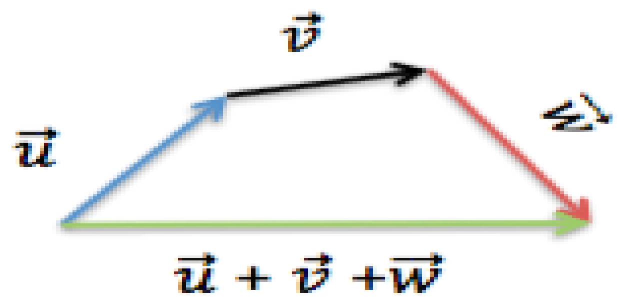
Operations sur les vecteurs /Operations on vectors : Somme de vecteurs/ Vector Addition

The sum of two vectors $\vec{u}$ and $\vec{v}$ is another vector $\vec{S}$ defined by

$\vec{S} = \vec{u} + \vec{v} = \vec{v} + \vec{u}$. So it is a **commutative** operation.



- vectorial sum of multiple vectors:

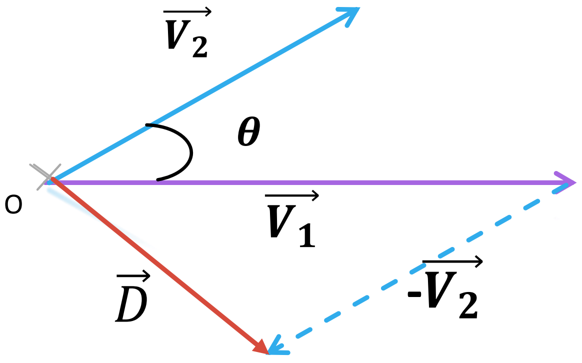


$$\vec{S} = \vec{\boldsymbol{v}} + \vec{u} + \vec{w} + \vec{z}$$

Subtraction of Vectors

Vector subtraction is not a commutative operation :

$$\vec{D} = \vec{u} - \vec{v} \neq \vec{v} - \vec{u}.$$



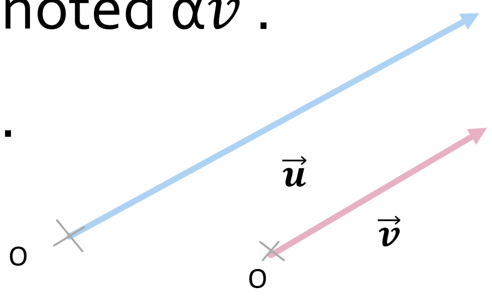
Scalar Multiplication

Vectors can be multiplied by scalars to change their magnitude. Multiplying a vector by a positive scalar scales its magnitude, while multiplying by a negative scalar reverses its direction.

- the Product of a vecteur $\vec{v}$ by a scalaire α is a vecteur noted $\alpha\vec{v}$.

Note that, $\alpha\vec{v}$ is also a vector, collinear to the vector $\vec{v}$.

$$\vec{u} = \alpha\vec{v}$$



Vector Components :

- Let A and B be two points in a Cartesian coordinate system

A(x_a , y_a) et B(x_b ,y_b) So the vector components $\overrightarrow{AB}$:

$$< x_b - x_a , y_b - y_a > \quad \text{ou bien:} \quad \begin{pmatrix} x_b - xa \\ y_b - yb \end{pmatrix}$$

- The vector $\overrightarrow{AB}$ is written as:

$$\overrightarrow{AB} = (x_b - x_a)\,\vec{i} + (y_b - y_a)\,\vec{j} = a\,\vec{i} + b\vec{j}$$

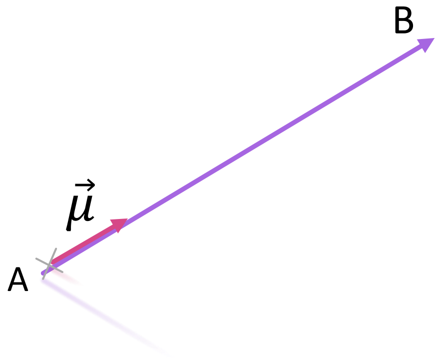
- The length (magnitude) or module of the vector $\overrightarrow{AB}$ is written:

$$\left|\overrightarrow{AB}\right| = \sqrt{a^2 + b^2}$$

Vecteur unitaire   Unit vectors

- The unit vector of the vector $\overrightarrow{AB}$ is obtained by dividing this vector by its magnitude

$$\vec{\mu} = \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|}$$



Unit vectors are vectors with a magnitude of 1 and are often used to specify direction. In three dimensions, the unit vectors along the x, y, and z axes are denoted as $\vec{i}$, $\vec{j}$, and $\vec{k}$, respectively.

I-2 Produit scalaire / Dot product

a/ Geometrical interpretation of scalar product

- Let $\vec{u}$ et $\vec{v}$ be two vectors forming a geometric angle θ, the real (scalar) number is called the dot product and is denoted as $\vec{u} \cdot \vec{v}$:

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cdot \cos\theta$$

It is the product of the magnitude of $\|\vec{v}\|$ By projecting $\vec{u}$ onto the direction of $\vec{v}$: $(\|\vec{u}\| \cos\theta)$.

b/ Expression analytique du Produit scalaire

Analytical expression of the Dot product

Let: $\vec{u} = <X, Y, Z>$ et $\vec{v} = <X', Y', Z'>$

The analytical expression of the scalar product $\vec{u} \cdot \vec{v}$ is given by:

$$\vec{u} \cdot \vec{v} = XX' + YY' + ZZ'$$

In fact:

$$(X\vec{i} + Y\vec{j} + Z\vec{k}).(X'\vec{i} + Y'\vec{j} + Z'\vec{k}) = XX' + YY' + ZZ'$$

Since :

$$\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1 \quad \& \quad \vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{k} = \vec{k} \cdot \vec{i} = \vec{k} \cdot \vec{j} = \vec{i} \cdot \vec{k} = 0$$

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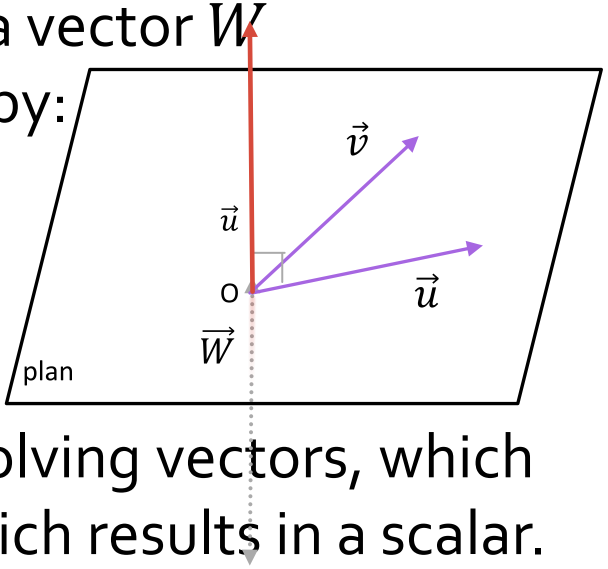
I-3 / Produit vectoriel / Cross product

a- Geometric expression of the Cross product

The Cross product of two vectors $\vec{u}$ and $\vec{v}$ noted $\vec{u} \wedge \vec{v}$ is a vector $\overrightarrow{W}$ perpendicular (orthogonal) to both of them, and defined by:



$$\vec{u} \wedge \vec{v} = |\vec{u}|.|\vec{v}\,|.\sin\theta$$

The cross product is another mathematical operation involving vectors, which results in a vector as its output, unlike the dot product which results in a scalar.

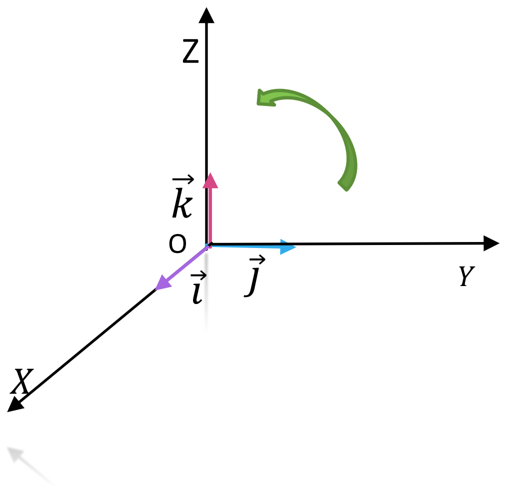
b/ Analytical expression of the cross product

$$\vec{u} \wedge \vec{v} = (x\vec{i} + y\vec{j} + z\vec{k}) \wedge (x'\vec{i} + y'\vec{j} + z'\vec{k})$$

$$= (yz' - zy')\vec{i} - (xz' - x'z)\vec{j} + (xy' - x'y)\vec{k}$$

Such as:

$$\vec{i} \wedge \vec{i} = \vec{j} \wedge \vec{j} = \vec{k} \wedge \vec{k} = \vec{0} \quad \text{et} \quad \vec{i} \wedge \vec{j} = \vec{k} \ ; \ \vec{j} \wedge \vec{k} = \vec{i} \ ; \ \vec{k} \wedge \vec{i} = \vec{j}$$

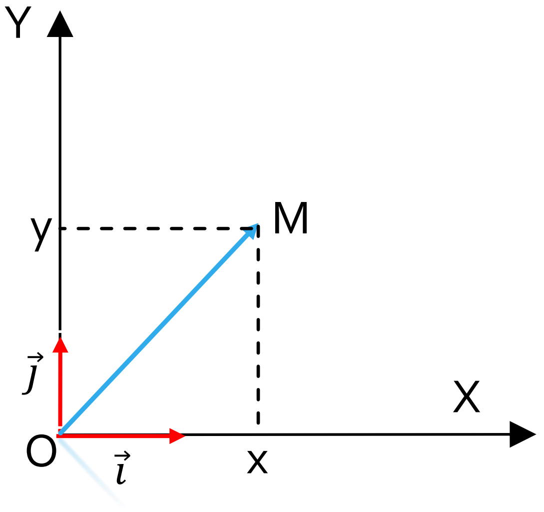


Méthode matricielle /Matrix method :

$$\vec{W} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ x' & y' & z' \end{bmatrix} = \vec{i}\begin{vmatrix} y & z \\ y' & z' \end{vmatrix} - \vec{j}\begin{vmatrix} x & z \\ x' & z' \end{vmatrix} + \vec{k}\begin{vmatrix} x & y \\ x' & y' \end{vmatrix}$$

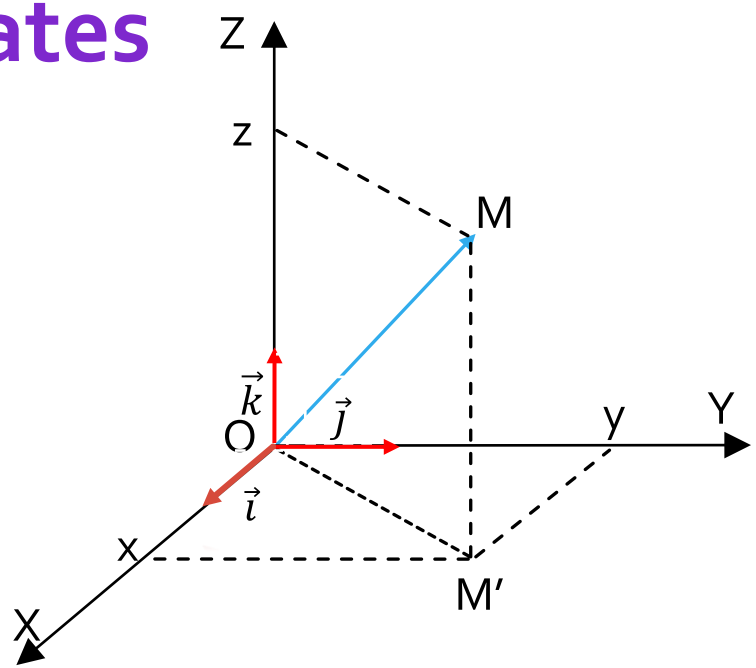
$$\vec{W} = (yz' - zy')\vec{i} - (xz' - x'z)\vec{j} + (xy' - x'y)\vec{k}$$

I-4 / Systèmes usuels de coordonnées
I-4 / Usual coordinate systems
a / Coordonnées Cartésiennes
a / Cartesian Coordinates



$$\overrightarrow{OM} = x\vec{i} + y\vec{j}$$

In the plan



$$\overrightarrow{OM} = x\vec{i} + y\vec{j} + z\vec{k}$$

In the space

b/ Coordonnées polaires
b/ Polar coordinates

M (r ,θ) ⟹ the polar base is $(\vec{e}_r, \vec{e}_\theta)$.

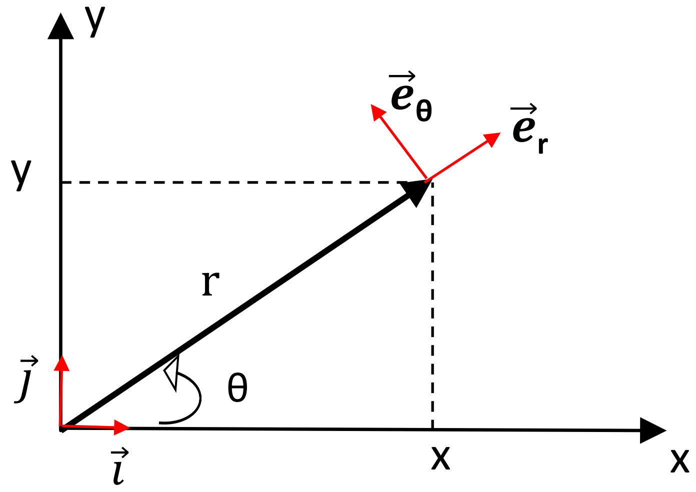
$$\overrightarrow{OM} = r\vec{e}_r$$

Polar coordinates are linked to Cartesian coordinates by:

$$x = r\cos\theta$$
$$y = r\sin\theta$$

⟹

$$r = \sqrt{x^2 + y^2}$$
$$\tan\theta = \frac{y}{x}$$



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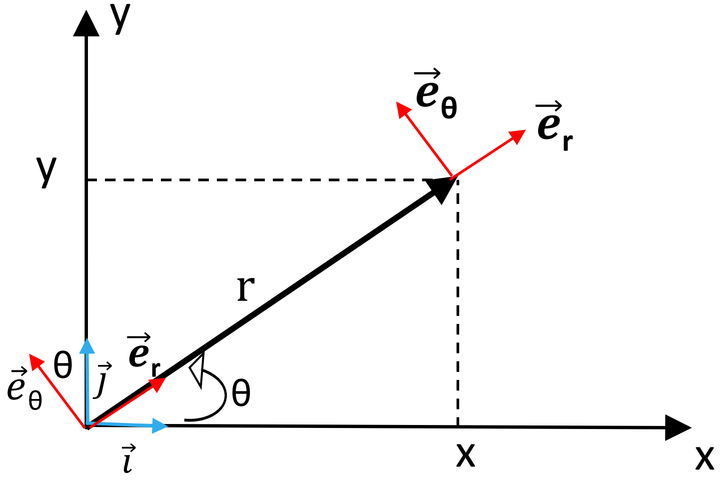
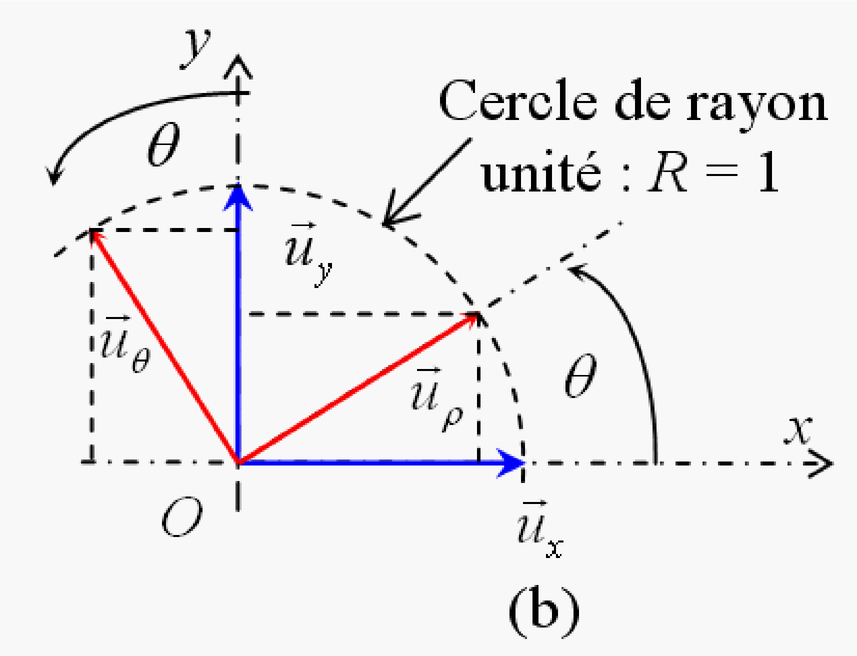
$$\vec{e}_r = \cos\theta \ \vec{i} + \sin\theta \ \vec{j}$$

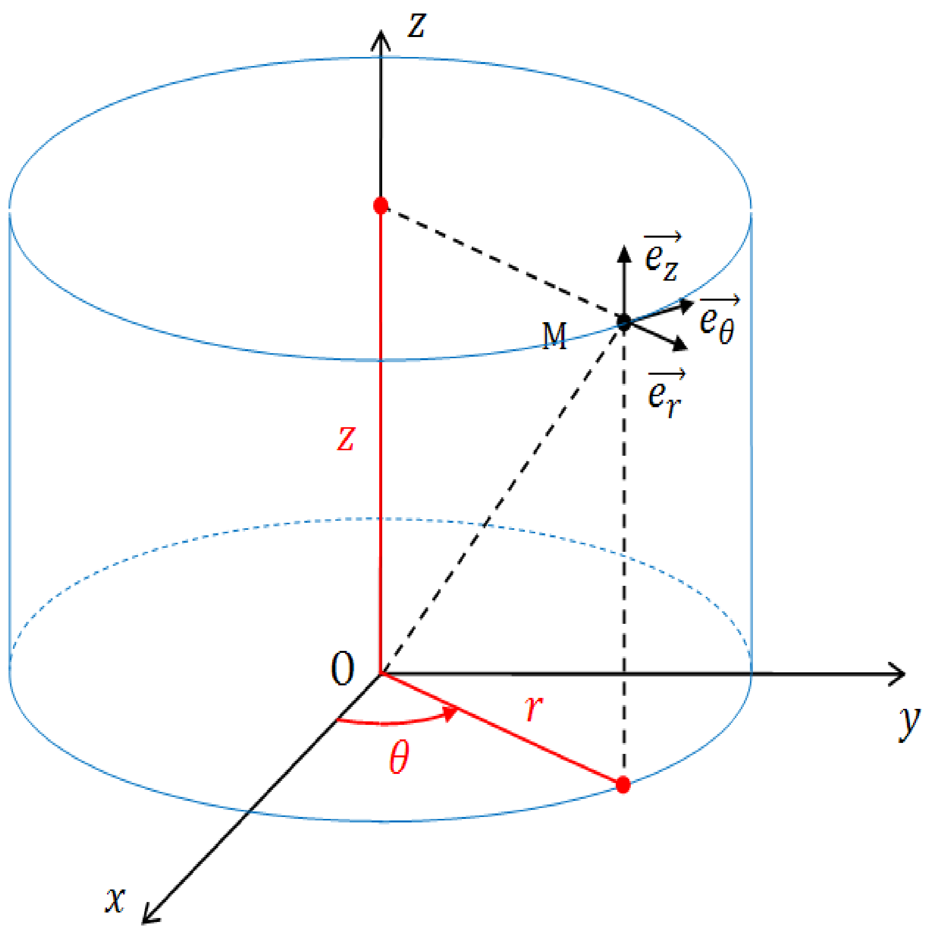
$$\vec{e}_\theta = -\sin\theta \ \vec{i} + \cos\theta \ \vec{j}$$





c / Coordonnées cylindriques
c / Cylindrical coordinates

M (r, θ, z) ⟶ The cylindrical base $(\vec{e}_r, \vec{e}_\theta, \vec{e}_z)$.



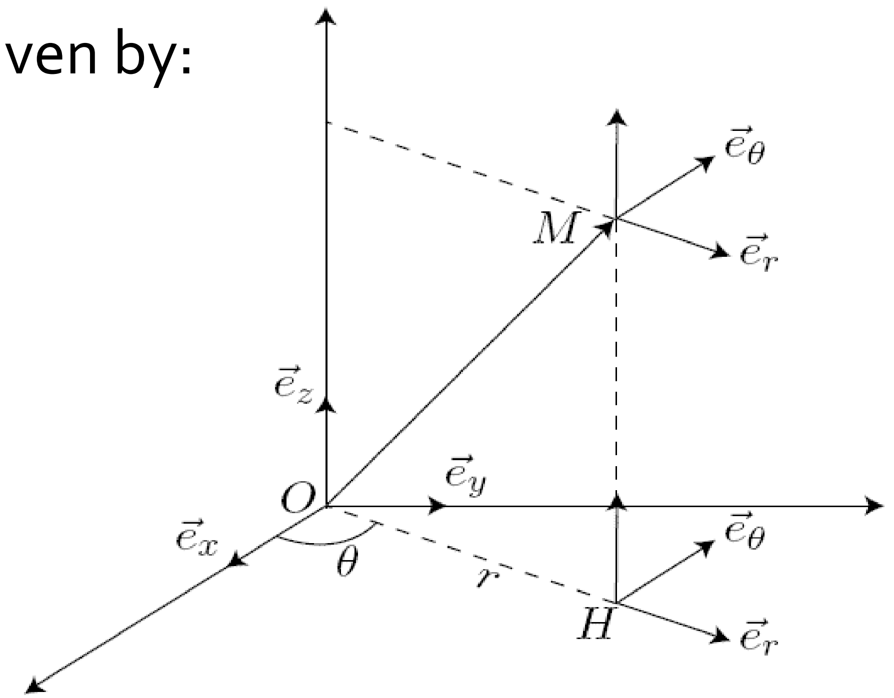
Cylindrical coordinates are linked to Cartesian coordinates by:

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases} \Rightarrow \begin{cases} r = \sqrt{x^2+y^2} \\ \tan\theta = \dfrac{y}{x} \\ z = z \end{cases}$$

The position vector in cylindrical coordinates is given by:

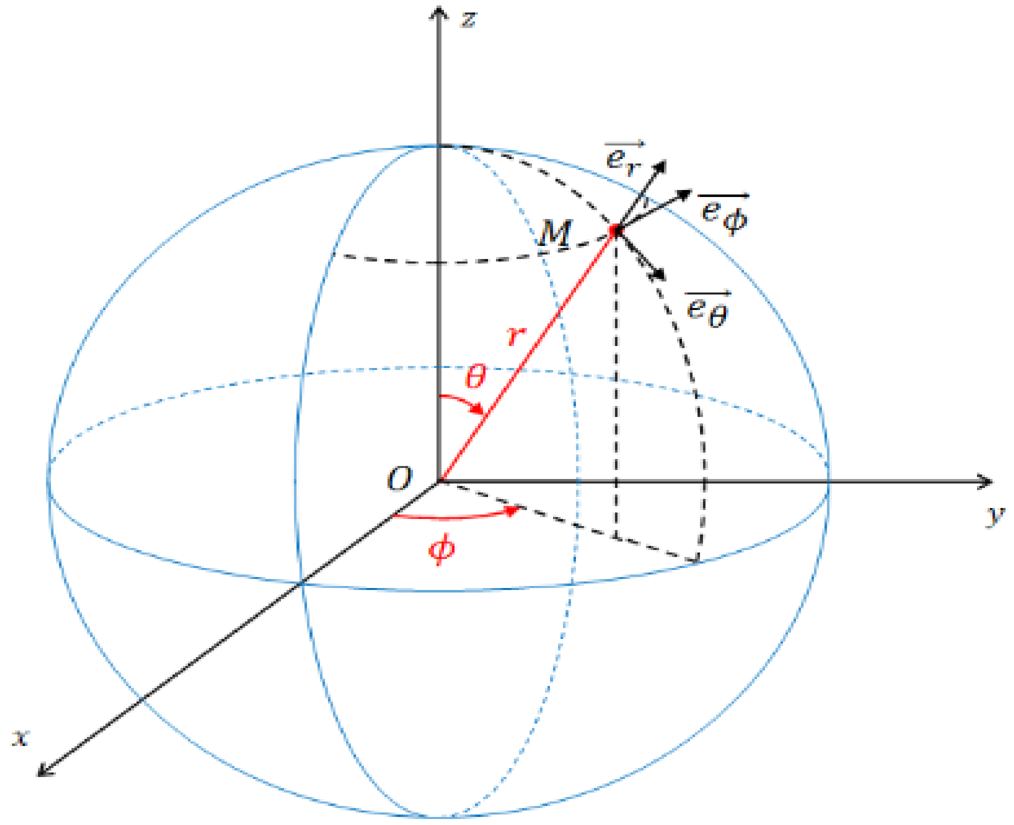
$$\overrightarrow{OM} = \overrightarrow{OH} + \overrightarrow{HM}$$

$$\overrightarrow{OM} = r\vec{e}_r + z\,\vec{e}z$$



d / Coordonnées sphériques
d / Spherical coordinates

M (r, θ, ϕ) ⟹ the spherical base is $(\vec{e}_r, \vec{e}_\theta, \vec{e}_\phi)$



$M\ (r, \theta, \phi)$ → the spherical base is $(\vec{e}_r, \vec{e}_\theta, \vec{e}_\phi)$.

Spherical coordinates are linked to Cartesian coordinates by :

$$\begin{cases} x = OH\cos\varphi \\ y = OH\sin\varphi \\ z = r\cos\theta \end{cases} \quad \text{avec } OH = r\sin\theta \quad \begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\varphi\sin\theta \\ z = r\cos\theta \end{cases}$$

The position vector of point M in the spherical base is given by :

$$\overrightarrow{OM} = r\vec{e}_r$$

Chapitre II Cinématique du point matériel
Chapter II: Kinematics of particles

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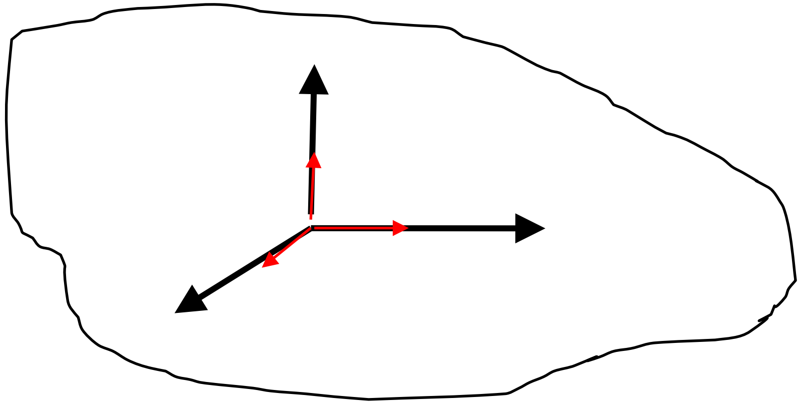
II-1 / Generalities

- The word 'kinematics' comes from the Greek word 'Cinema,' which means movement.
- Kinematics is the study of the motion of a solid, determining its position, velocity, and acceleration.
- Kinematics is the branch of mechanics that examines and describes the motion of an object considered infinitesimally small, referred to as a point particle, denoted as M, with a mass denoted as m.

II-2 / / Locating a Mobile

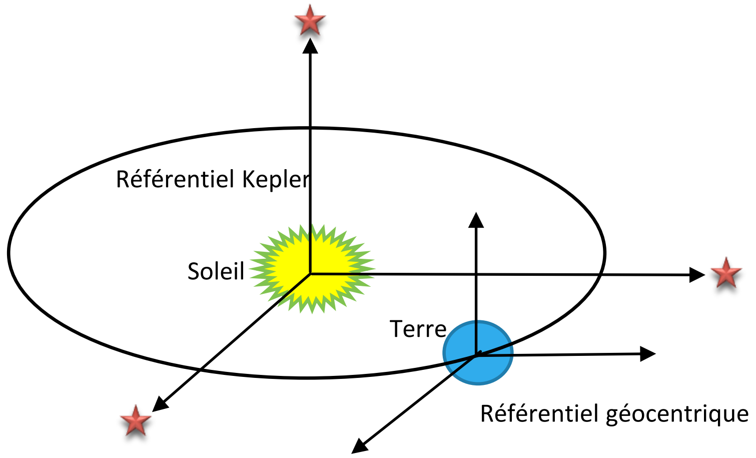
- The set of points described by point M over time is called its trajectory.
- On its trajectory, point M has a velocity vector $\vec{V}$ and an acceleration vector $\vec{a}$.
- To study the motion of a point, we establish a frame of reference, and for that purpose, we define a reference frame or space.

Point M could be an airplane, for example, and thus we can locate the position of our airplane using the position vector $\overrightarrow{OM}$.



- A reference frame is defined as a set of points whose distances remain constant over time. It is typically characterized by a point O, chosen as the origin of the frame conventionally, and equipped with an orthonormal basis.
- To define the position of a point in space, an observer will use a reference frame, a coordinate system linked to a clock to measure time.
- This space-time reference frame is called a "reference frame" or simply "frame of reference”.

- The Earth reference frame is the most commonly used reference frame: it is centered at a point on Earth, and its axes are tied to the Earth's rotation.
- The geocentric reference frame has its origin at the center of mass of the Earth, and its axes are defined with respect to three stars distant enough to appear stationary. Therefore, it is not fixed to the Earth in its rotational motion around its poles.
- The Keplerian reference frame (or heliocentric) is a reference frame centered on the center of mass of the Sun, with its axes parallel to those of the Copernican reference frame.



a/ Position vector

- The position vector is expressed in various coordinate systems as follows:
- Cartesian Coordinates $\overrightarrow{OM} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$
- Polair Coordinates $\overrightarrow{OM} = r\vec{e}_r$
- Cylindrical Coordinates $\overrightarrow{OM} = r\vec{e}_r + z\,\vec{e}_z$
- Spherical Coordinates $\overrightarrow{OM} = r\vec{e}_r$

The trajectory equation

- The mathematical relationship that connects the coordinates independently of time is called the trajectory equation.
- The path followed by the particle in space as it moves is called its trajectory. It can be described using equations of motion.
- y = f(x) ou r = f(Ө)
- Example of equation of a cercle is : $x^2 + y^2 = R^2$
- Parametric equations or Time equations : It is the relationships that provide us distances as functions of time.

X =f(t) ; y = g(t) ; z = h(t);

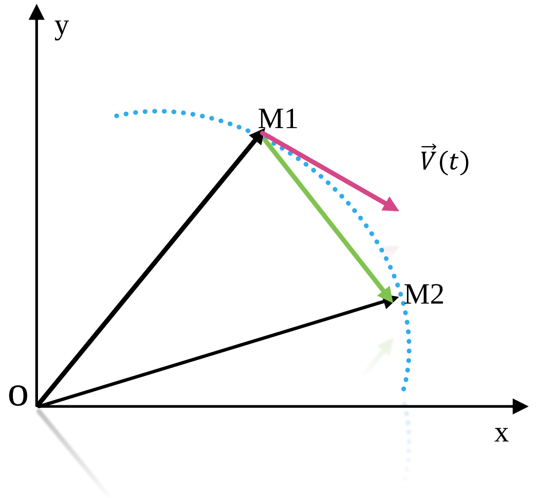
b/ Velocity vector

- Velocity is a vector quantity that provides information about the change in position of a point with respect to time. Velocity is the rate of change of position with respect to time. with units of m/s (meters per second).
- By analyzing the velocity vector, you can determine the direction and sense of motion, while the displacement vector provides information about the magnitude of the change in position. These concepts are fundamental in understanding the kinematics of a particle's motion.
- It must express the instantaneous direction of the point's displacement, the sense of the displacement, and the magnitude of the variation of this displacement.
- Velocity is a vector quantity, its direction is tangent to the trajectory.

Average velocity:

is the displacement of an object divided by the time:
It is the ratio of the displacement to the time it takes to cover that displacement.

$$\vec{V}m = \frac{\overrightarrow{M1M2}}{t2-t1} = \frac{\overrightarrow{OM2} - \overrightarrow{OM1}}{\Delta t}$$

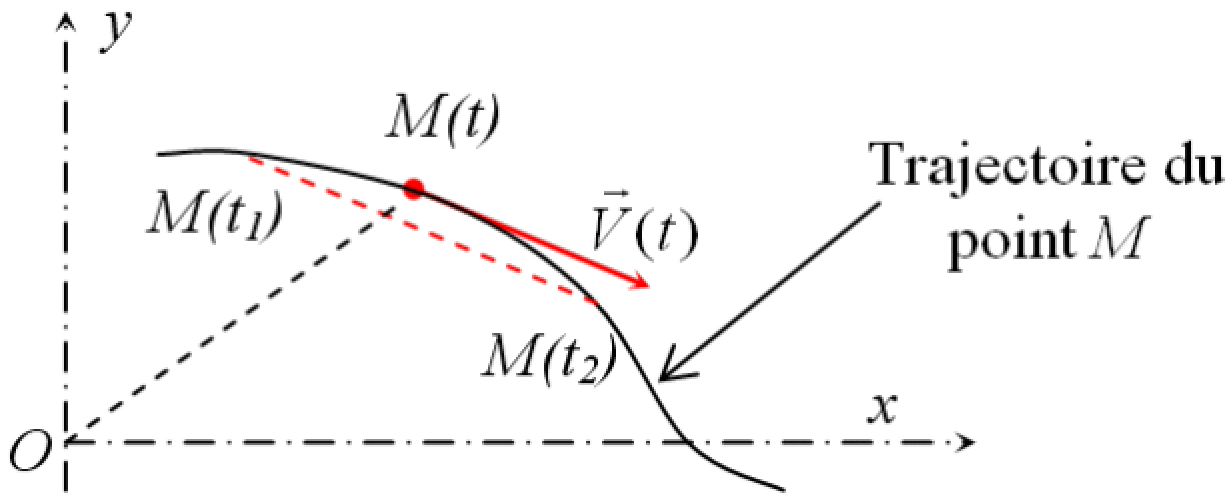


Instantaneous velocity: It is the velocity at a specific moment t. It can be defined as the average velocity between the position M1 of the point at time t and the position M2 of the same point at time (t + Δt), where Δt represents a very small duration.

$$\vec{V}(t) = \lim_{\Delta t \to 0} \frac{\overrightarrow{OM}(t+\Delta t) - \overrightarrow{OM}(t)}{\Delta t} = \frac{d\overrightarrow{OM}}{dt}$$

We note that the velocity is the rate of change of position with respect to time

The average velocity : of a point M approaches the instantaneous velocity at time t as Δt approaches 0. As M2 approaches M1, the chord M1M2 approaches the tangent to the trajectory at point M, from which the velocity vector becomes a tangent vector to the trajectory at the point in question.



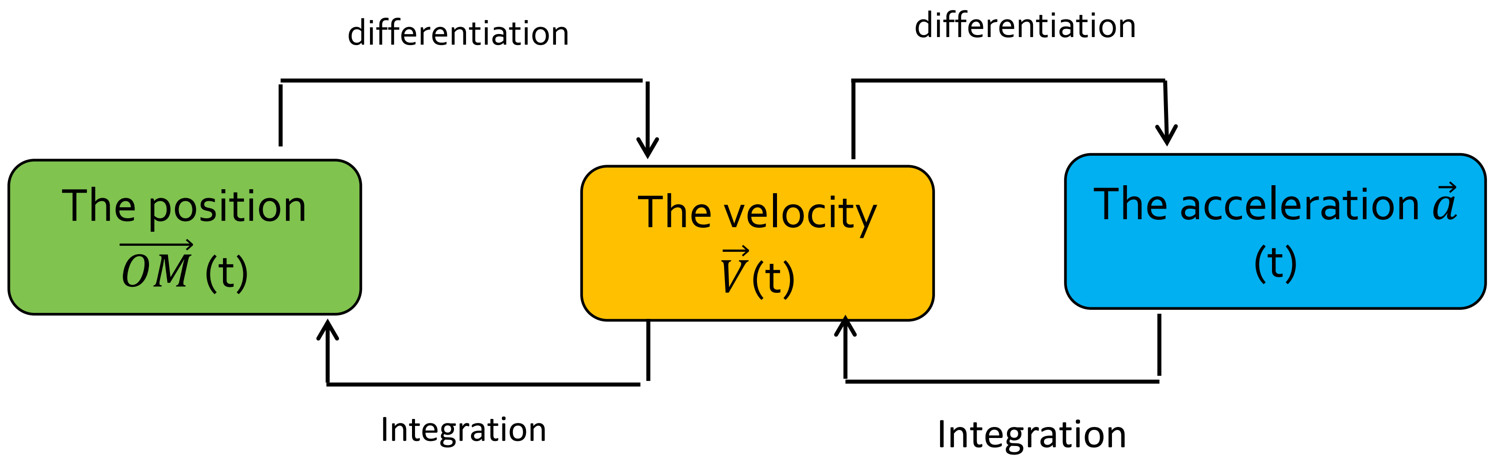
(Path followed by the point M)

c/ acceleration vector

- Just as the velocity vector informs us about the change in the position vector over time, the acceleration vector informs us about the changes in the velocity vector over time.
- The acceleration vector represents the first derivative with respect to time of the velocity vector or the second derivative of the position vector.

$$\vec{a}(t) = \lim_{t' \to t} \frac{\vec{v}' - \vec{v}}{t' - t} = \lim_{t' \to t} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt};\ \vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d^2\overrightarrow{OM}}{dt^2}$$

The three kinematic equations, which are the position vector $\overrightarrow{OM}$ (t), the velocity $\vec{V}$ (t) and the acceleration $\vec{a}$ (t), are mathematical functions that can be derived from each other through differentiation and integration.



II- 3 / Expression of the velocity vector in Cartesian coordinates

- Le vecteur vitesse en coordonnées cartésiennes est donné par :

$$\vec{V}(\text{t}) = \frac{d\overrightarrow{OM}}{dt} = \frac{d}{dt}(\text{x}\vec{\imath} + \text{y}\vec{\jmath} + \text{z}\vec{k})$$

$$\vec{V}(\text{t}) = \frac{dx}{dt}\vec{\imath} + \frac{dy}{dt}\vec{\jmath} + \frac{dz}{dt}\vec{k}$$

$$\vec{V}(\text{t}) = \text{Vx}\,\vec{\imath} + \text{VY}\,\vec{\jmath} + \text{VZ}\,\vec{k} \qquad \vec{V}(\text{t}) = \dot{X}\,\vec{\imath} + \dot{Y}\,\vec{\jmath} + \dot{Z}\vec{k}$$

- Velocity in Cartesian coordinates is the rate of change of position with respect to time. It can be expressed as the vector (vx, vy, vz), where each component represents the rate of change along the respective axis.
- Since the Cartesian basis is a fixed basis over time, its unit vectors are independent of time, and their derivative with respect to time is zero.

II-4 /Expression of the acceleration vector in Cartesian coordinates

- The acceleration vector in Cartesian coordinates is given by:

$$\vec{a}(t) = \frac{d\vec{V}}{dt} = \frac{d}{dt}(Vx\,\vec{i} + Vy\,\vec{j} + Vz\,\vec{k})$$

Since the Cartesian basis is a fixed basis over time, its unit vectors are independent of time, and their derivative with respect to time is zero.

$$\vec{a}(t) = ax\,\vec{i} + aY\,\vec{j} + aZ\,\vec{k} \qquad \vec{a}(t) = \ddot{X}\,\vec{i} + \ddot{Y}\,\vec{j} + \ddot{Z}\,\vec{k}$$

II-5/ Expression of the acceleration vector in polar coordinates

- $\vec{V}(t) = \frac{d\overrightarrow{OM}}{dt}$ with: $\overrightarrow{OM}$ in polar coordinates $\overrightarrow{OM} = r\vec{e}_r$
- $\vec{V}(t) = \frac{d}{dt}(r\vec{e}_r) = \dot{r}\,\vec{e}_r + r\frac{d}{dt}\vec{e}r$

Mathematical Reminder :

The rule for the derivative of a composite function is:

$$f = f(y) \text{ et } y = f(x)$$

$$\frac{df}{dx} = \frac{df}{dy} \cdot \frac{dy}{dx}$$

In our case, y represents the angle θ, and x represents time

SO : $\frac{d}{dt}\vec{e}_r = \frac{d}{d\theta}\vec{e}_r \,.\, \frac{d\theta}{dt} = \dot{\theta}\ \frac{d}{d\theta}\vec{e}_r$

- $\vec{e}_r = \cos\theta\ \vec{i} + \sin\theta\ \vec{j}$; $\frac{d\vec{e}r}{d\theta} = -\sin\theta\ \vec{i} + \cos\theta\ \vec{j} = \vec{e}_\Theta$
- $\vec{e}_\Theta = -\sin\theta\ \vec{i} + \cos\theta\ \vec{j}$; $\frac{d\vec{e}_\theta}{d\theta} = = -\cos\theta\ \vec{i} - \sin\theta\ \vec{j} = -\vec{e}_r$
- $\frac{d}{dt}\vec{e}_r = \dot{\theta}\ \vec{e}_\Theta$ et : $\frac{d}{dt}\vec{e}_\theta = -\dot{\theta}\ \vec{e}_r$

- So, the expression for velocity in polar coordinates becomes:

$$\vec{V}(t) = \frac{d}{dt}(r\vec{e}_r) = \dot{r}\ \vec{e}_r + r\frac{d}{dt}\vec{e}_r$$

$$\boldsymbol{\vec{V}(t) = \dot{r}\ \vec{e}_r + r\dot{\theta}\ \vec{e}_\Theta}$$

II-6/Expression of the acceleration vector in polar coordinates

- $\vec{a}(t) = \frac{d\vec{V}}{dt} = \frac{d}{dt}(\dot{r}\ \vec{e}_r + r\,\dot{\theta}\,\vec{e}_\Theta)$
- $\vec{a}(t) = \ddot{r}\,\vec{e}_r + \dot{r}\,\frac{d}{dt}\vec{e}_r + \dot{r}\,\dot{\theta}\,\vec{e}_\Theta + r\,\ddot{\theta}\,\vec{e}_\Theta + r\,\dot{\theta}\,\frac{d}{dt}\vec{e}_\theta$
- $\vec{a}(t) = (\ddot{r} - r\,\dot{\theta}^2)\,\vec{e}_r + (2\dot{r}\,\dot{\theta} + r\,\ddot{\theta})\,\vec{e}_\theta$

II-7/ expressions of the position , velocity and acceleration vector in cylindrical coordinates :

Position vector : $\overrightarrow{OM} = r\vec{e}_r + z\,\vec{e}_z$ ou $\vec{e}_z = \vec{k}$

Velocity vector : $\vec{V}(t) = \dfrac{d\overrightarrow{OM}}{dt}$

$\rightarrow \quad \vec{V}(t) = \dfrac{d}{dt}(r.\vec{e}_r + z\vec{k}) = \dot{r}\,\vec{e}_r + r\dfrac{d}{dt}\vec{e}r + \dot{z}\,\vec{k}$

With : $\dfrac{d}{dt}\vec{e}_r = \dot{\theta}\,\vec{e}_\theta$ *et* $\dfrac{d}{dt}\vec{e}_\vartheta = -\dot{\theta}\,\vec{e}_r$

So the expression of the velocity in cylindrical coordinates is :

$$\vec{V}(t) = \dot{r}\,\vec{e}_r + r\dot{\theta}\,\vec{e}_\theta + \dot{z}\,\vec{k}$$

Acceleration Vector :

$$\vec{a}(t) = \frac{d\vec{V}}{dt} = \frac{d}{dt}(\dot{r}\ \vec{e}_r + r\,\dot{\theta}\ \vec{e}_\theta) = \ddot{r}\ \vec{e}_r + \dot{r}\ \frac{d}{dt}\vec{e}_r + \dot{r}\ \dot{\theta}\ \vec{e}_\theta + r\,\ddot{\theta}\ \vec{e}_\theta + r\,\dot{\theta}\ \frac{d}{dt}\vec{e}_\vartheta + \ddot{z}\ \vec{k}$$

$$\vec{a}(t) = (\ddot{r} - r\,\dot{\theta}^2)\,\vec{e}_r + (2\dot{r}\ \dot{\theta} + r\,\ddot{\theta})\,\vec{e}_\vartheta + \ddot{z}\vec{k}$$

II-8/Repère de Frenet
II-8/ Frenet frame

It is an orthonormal moving frame (M, T, N)

T Being a unit vector tangent to the trajectory.

N Being a unit vector normal to the trajectory, directed towards the center of curvature of the trajectory. At every point on the trajectory, one can define a circle (locally, a segment of the curve always resembles, more or less, a circle) with a radius R, which is the radius of curvature of the trajectory.

$\vec{V} = V\,\vec{u}_T$	$\vec{V}$ **is tangent to the trajectory**
$\vec{a} = \frac{d\vec{V}}{dt} = \frac{dv}{dt}\,\vec{u}_T + \frac{v^2}{R}\,\vec{u}_N$	$\vec{a}$ is directed towards the center of curvature of the trajectory.

- $a_T = \frac{dv}{dt}$ is the value of the tangential acceleration
- It can be positive, negative, or zero.
- $a_N = V^2/R$ is The value of the normal acceleration.
- It can be positive or zero.
- The radius of curvature of the trajectory is denoted by R.
- $\vec{a} = \vec{a}_n + \vec{a}_t$ d'où $a^2 = a_n{}^2 + a_t{}^2$

Chapitre III : Etude des mouvements usuels

Chapter III : Study of usual movements

III-1 / Mouvement rectiligne	Rectilinear motion
III-1–a/ Mouvement rectiligne uniforme (MRU)	Uniform rectilinear motion (URM)
III-1–b/ Mouvement rectiligne uniformément varié	Uniformly varied rectilinear motion
III-2 / Mouvement circulaire	Circular movement
III-2–a/ Mouvement circulaire uniforme	Uniform circular motion
III-2–b/ Mouvement circulaire uniformément varié	Uniformly varied circular motion
III-3 / Mouvement rectiligne sinusoidal	Sinusoidal rectilinear motion
III-4 / Mouvement parabolique	Parabolic movement

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III-1 Mouvement Rectiligne : Rectilinear motion

A material point M is in rectilinear motion if its trajectory is a straight line along a single axis of the reference frame $(O, \vec{i}, \vec{j}, \vec{k})$, where the motion of point M takes place. Therefore, we only need a single parameter, such as 'ox,' to define the position of point M.

III-1-a/Mouvement rectiligne uniforme (MRU):Uniform rectilinear motion (URM)

A material point is in uniform rectilinear motion if its trajectory is a straight line and its velocity vector is constant. V=Vo=$\dot{X}$ = constant, so the acceleration vector is zero (a = $\frac{dv}{dt}$ = 0).

Equation horaire du mouvement : Equation of motion

Equations of Motion: To describe uniformly rectilinear motion, we can use a set of equations that relate the initial velocity, final velocity, acceleration, displacement, and time. The most commonly used equation is:

We choose the X-axis as the rectilinear reference.

$$V=V_0=\dot{X} = \text{constante donc } \vec{V}(t) = V_0\vec{\imath} \;\; \& \; a = 0$$

$$\rightarrow V = V_0 = \frac{dx}{dt} \rightarrow dx = V_0 dt \Rightarrow \int_{x_0}^{x} x = \int_{0}^{t} V_0 dt$$

The equation of motion for uniform rectilinear motion is:

$$\Rightarrow \quad \boldsymbol{x(t) = V_0 t + x_0}$$

With $\boldsymbol{x_0}$ being an integration constant determined from the initial conditions..

Diagrammes du mouvement(MRU)/ Motion Diagrams

These diagrams are a way to visually represent the motion of objects or particles over time. Motion diagrams can be particularly useful in physics to help understand and analyze the behavior of objects in motion. There are several types of motion diagrams:

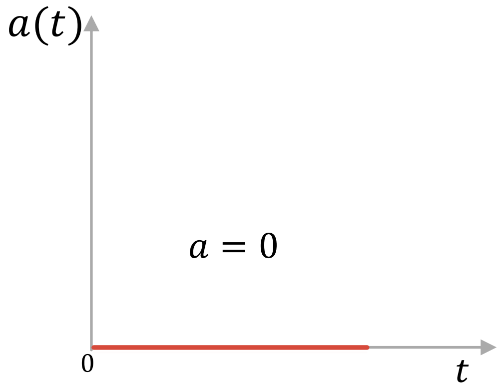
1/ Displacement-Time Diagram (Position-Time Diagram)

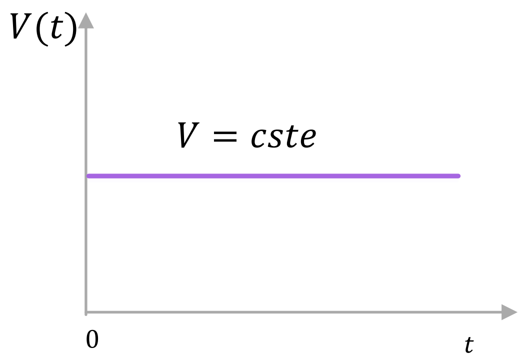
2/ Velocity-Time Diagram (Speed-Time Diagram)

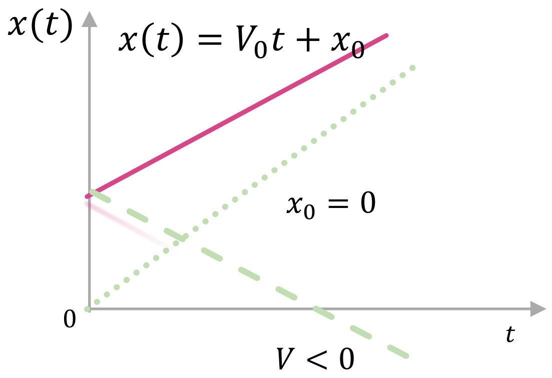
3/ Acceleration-Time Diagram

(MRU) : The accleration : a = 0 m/s^2, - The velocity : $V=V_0$= cste

the displacement as a function of time: $x(t) = V_0 t + x_0$







III-1-b Mouvement rectiligne uniformément varie (MRUV): Uniformly varied rectilinear motion(UVRM)

- The motion of a material point is rectilinear uniformly varied if its trajectory is a straight line, and its acceleration is constant. $\vec{a} = a_0\vec{\imath} = cste$
- Considering the initial conditions:(t=o, V(o)= V_0).
- $\vec{a} = \frac{d\vec{V}}{dt} \Rightarrow d\vec{V} = a_0\vec{\imath}\,dt \Rightarrow \int_{V_0}^{V} d\vec{V} = \int_0^t a_0\vec{\imath}\,dt$
- $\Rightarrow \vec{V}(t) = (a_0\,t + V_0)\vec{\imath}$
- This yields the equation for instantaneous velocity : $V(t) = (a_0\,t + V_0)$

Equation horaire du mouvement : Equation of motion as a function of time.

- Considering the initial conditions: (pour t=o, $x(0) = x_0$ et V(o) = V_0).

And considering the equation :

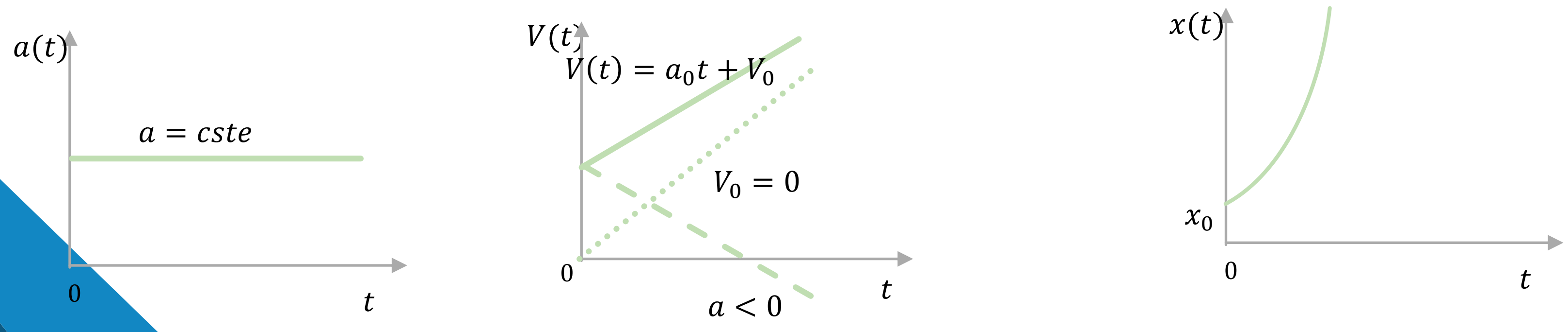
$$V(t) = \frac{dx(t)}{dt} \Rightarrow \int_{x_0}^{x} dx(t) = \int_0^t V(t)dt \qquad = \int_0^t (a_0\, t + V_0)dt$$

- The equation of Uniformly varied rectilinear motion is(UVRM) :

- $x(t) = (\frac{1}{2}a_0 t^2 + V_0 t + x_0)$

So the equations of UVRM are : $\begin{cases} a = a_0 = cste \\ V(t) = (a_0\, t + V_0) \\ x(t) = \frac{1}{2}a_0 t^2 + V_0 t + x_0 \end{cases}$

The diagrams for uniformly accelerated rectilinear motion related to acceleration, velocity, and displacement are represented in the following figure:



The uniformly accelerated rectilinear motion is either accelerated or decelerated (retarded).

The motion is uniformly accelerated if the dot product $\vec{v} \cdot \vec{a} > 0$ is positive

The motion is uniformly decelerated if the dot product $\vec{v} \cdot \vec{a} < 0$ is negative

The sign of the acceleration vector $\vec{a}$ alone is not sufficient.

It is possible to obtain a relationship between position, velocity, and acceleration independent of time.

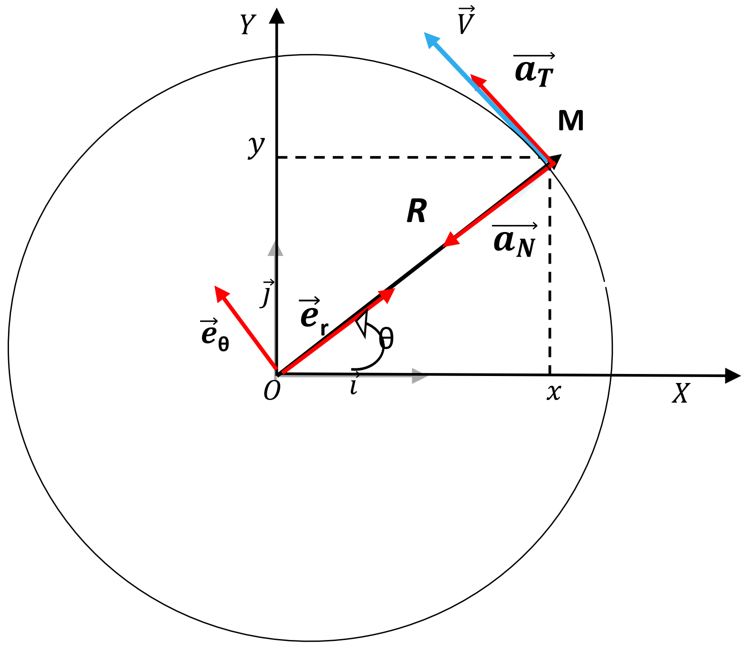
$$V = a_o t + v_o \Rightarrow t = (v - v_o) / a_o$$

By replacing t with its expression in the equation for position x(t), we obtain:

$$2 a_o (x - x_o) = v_f^2 - v_0^2$$

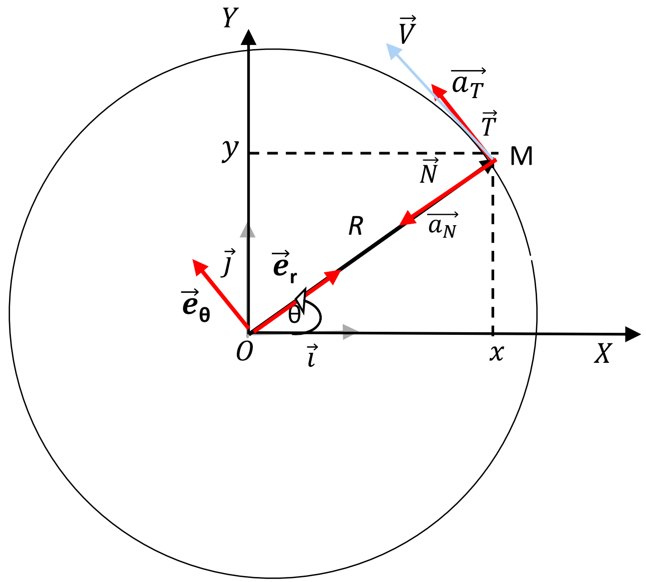
Mouvement circulaire / circular motion

- In circular motion, the trajectory of point M is a circle with center O and radius r. It is logical to choose the origin of the coordinate system as the center O of the circle, and the polar coordinate system is well-suited for this type of motion.With r = constant and θ = f(t)
- The equations of motion :
- $\overrightarrow{OM} = r\vec{e}_r$
- $\vec{V}(t) = \frac{d}{dt}(r\vec{e}_r) = r\frac{d}{dt}\vec{e}_r = r\,\dot{\theta}\,\vec{e}_\theta$
- $\vec{a}(t) = -\,r\,\dot{\theta}^2\,\vec{e}_r + r\,\ddot{\theta}\,\vec{e}_\theta$



Comparaison entre les deux expressions de l'accélération du MC dans la base de Frenet et dans la base polaire.
Comparison between the two expressions of the acceleration of the CM in the Frenet basis and in the polar basis.

Frenet basis : $\vec{a} = a_T.\vec{T} + a_N.\vec{N}$



- $a_T = \frac{dv}{dt}$ et $a_N = \frac{v^2}{R}$ Avec : $\vec{N} = -\overrightarrow{e_r}$ et $\vec{T} = \overrightarrow{e_\theta}$
- CM polar basis : $\vec{a} = \mathrm{R}\ddot{\theta}\overrightarrow{e_\theta} - R\dot{\theta}^2\overrightarrow{e_r}$

By identification :

- $a_T = \mathrm{R}\ddot{\theta}$ et $a_N = R\dot{\theta}^2$
- or $\vec{V} = R\dot{\theta}\overrightarrow{e_\theta} = R\omega\overrightarrow{e_\theta}$ & $V = R\omega \rightarrow \omega = \frac{V}{R}$
- $\frac{dV}{dt} = R\dot{\omega} = R\ddot{\theta} \rightarrow a_T = \frac{dV}{dt}$ et
- $a_N = R\left(\frac{V}{R}\right)^2 = \frac{V^2}{R}$.

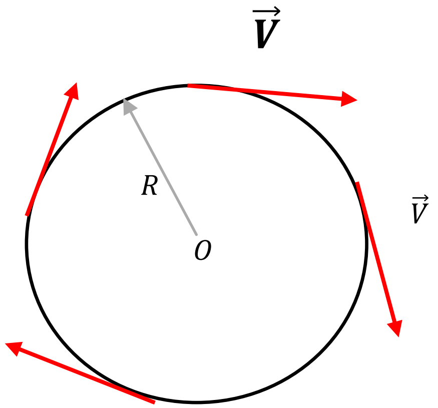
Mouvement circulaire uniforme / Uniform Circular Motion

- The circular motion is uniform if the angular speed is constant $w = \dot{\theta}$ = Cste

$$\overrightarrow{OM} = r\vec{e}_r$$

$$\vec{V}(t) = r\,\dot{\theta}\,\vec{e}_\theta = r\,w\,\vec{e}_\theta$$

$$\vec{a} = -\,r\dot{\theta}^2\,\vec{e}_r$$



$$w = \dot{\theta} = Cste \Rightarrow \theta = wt + \theta_o = \dot{\theta}t + \theta_o$$

With the angular velocity ω being constant, the tangential component of the acceleration vector is zero, leaving only the normal component. It is the normal component that 'causes rotation,' meaning it informs us about changes in the direction of the velocity vector, not its magnitude, which remains constant. Therefore, even in uniform motion (with constant V and ω), this acceleration necessarily exists.

Mouvement circulaire uniformément varié

A circular motion is uniformly accelerated if the angular acceleration is a constant.
$\ddot{\theta}$ = Cste

$$\begin{cases} \overrightarrow{OM} = R\,\overrightarrow{e_r} \\ \vec{V} = \dfrac{d\overrightarrow{OM}}{dt} = \mathrm{R}\dfrac{d\,\overrightarrow{e_r}}{dt} = R\dot{\theta}\overrightarrow{e_\theta} \\ \vec{a} = -R\dot{\theta}^2\overrightarrow{e_r} + \mathrm{R}\ddot{\theta}\overrightarrow{e_\theta} \end{cases}$$

With $\dot{\theta} = \ddot{\theta}0\,t + \dot{\theta}0$

$$\theta = \frac{1}{2}\ddot{\theta}0\,t2 + \dot{\theta}0\,t + \theta0$$

A uniformly accelerated circular motion is either accelerated or decelerated.

UCAM (Uniformly Accelerated Circular Motion) if the dot product $\dot{\theta}\,.\ddot{\theta}$ >o

UCRM (Uniformly Decelerated Circular Motion) if the dot product $\dot{\theta}\,.\ddot{\theta} < 0$

Mouvement Rectiligne Sinusoïdal/ Simple Harmonic Motion(SHM)

- The motion of a material point is rectilinear sinusoidal if its equation of motion can be written in the form: $\boldsymbol{x(t) = X_m \sin(\omega t + \varphi)}$

ou $\boldsymbol{x(t) = X_m \cos(\omega t + \varphi)}$

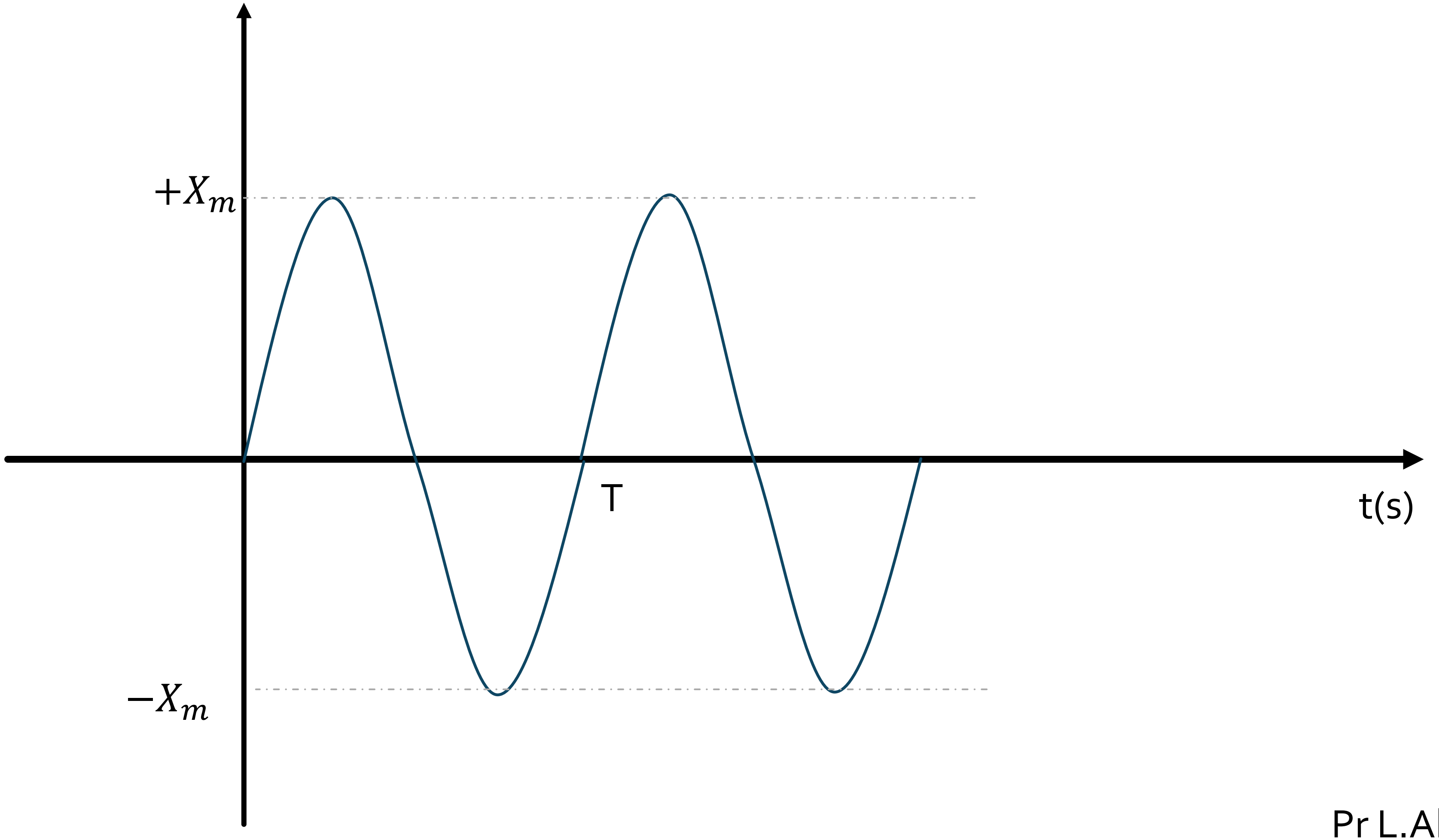
- X_m: Amplitude .
- X : instantaneous position, it varies between two extreme values $-X_m\ et\ +X_m$.
- ω : Pulsation of the motion, its unit is radians per second.
- φ: Initial phase, its unit is radians.
- $(\omega$t + $\varphi)$: Instantaneous phase, its unit is radians.

- **The velocity:** By differentiating the time equation, we obtain the expression for instantaneous velocity:
- $\boldsymbol{v(t) = \frac{dx(t)}{dt} \quad = \quad \dot{x}(t) = \omega X_m \cos(\omega t + \varphi)}$
- The velocity varies between two extreme values : $\pm\, \omega X_m$
- **The acceleration** :By differentiating the velocity we obtain the expression for instantaneous acceleration :
- $\boldsymbol{a(t) = \frac{dv(t)}{dt} = \ddot{x}(t) = -\omega^2 X_m \sin(\omega t + \varphi)}$
- $\boldsymbol{\ddot{x}(t) = -\omega^2 x(t)}$ d'où $\boldsymbol{\ddot{x}(t) + \omega^2 x(t)}$ = 0
- The period T is the constant time interval that separates two consecutive passages of the mobile at the same point.
- A motion is said to be periodic when it repeats itself exactly at identical time intervals (the period).

- From time t to t + T, the phase has increased by 2π and retains its value.
- $W[(t + T) + \varphi] = Wt + \varphi + 2\pi \Rightarrow WT = 2\pi \Rightarrow T = \frac{2\pi}{W}$ (second)
- The frequency f is the number of oscillations in one second.
- $f = \frac{1}{T}$ (hertz)
- This implies that: $W = 2\pi f$

SHM is a fundamental form of periodic motion where an object moves back and forth along a straight line or oscillates around an equilibrium position.

The trajectory is a sine or cosine wave, resulting in a smooth, repetitive motion.



Mouvement parabolique: Parabolic motion

- We launch a projectile M into the air with an initial velocity $\vec{V}_0$ Making an angle α with the horizontal (ox), its motion takes place in the (xoy) plane, and its trajectory is parabolic.
- To study the motion of M, we determine: its acceleration, velocity, position, and trajectory y=f(x).
- Projectile motion is the motion of an object that is thrown or projected into the air, moving under the influence of gravity. It follows a curved path, known as a trajectory. This motion can be broken down into two components: horizontal motion and vertical motion.

We decompose the motion of M along the two axes ox and oy:

- On ox (Horizontal Motion): The horizontal motion of a projectile is typically uniform and unaffected by gravity. This means that there is no horizontal acceleration. The object moves at a constant horizontal velocity:

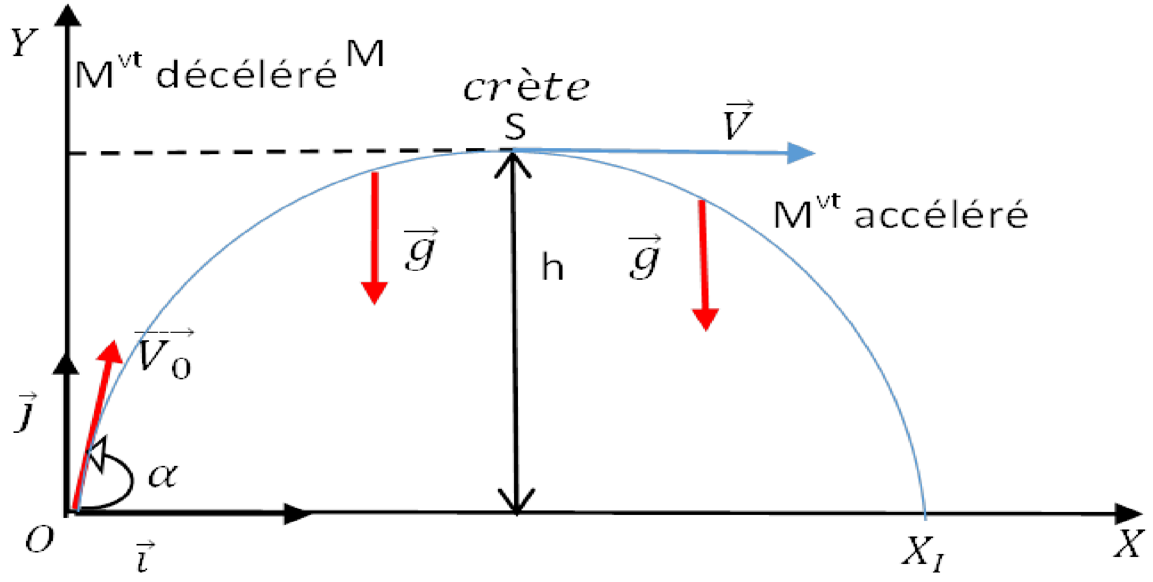
$$a_x = 0 \Rightarrow V_0 = cste \Rightarrow x(t) = V_{0x}\, t + x_0$$

URM On ox

- On oy(Vertical Motion): The vertical motion of the projectile is influenced by gravity. The object is subject to an acceleration due to gravity, which is typically -9.81 m/s² on Earth. This acceleration causes the object to accelerate downward.

$$a_y = -g \Rightarrow y(t) = \frac{1}{2} a t^2 + V_{0y}\, t + y_0$$

UVRM on oy



Y
M^vt décéléré M
crète
S
$\vec{V}$
M^vt accéléré
$\vec{g}$
h
$\vec{g}$
$\overrightarrow{V_0}$
$\vec{j}$
α
O
$\vec{i}$
X_I
X

•The time equations of motion:

ON ox :

$$a_x = \ddot{x} = 0 \Rightarrow \frac{dV_x}{dt} = 0 \Rightarrow dV_x = 0$$

$$\Rightarrow V_x = cste = C_1$$

$$V_x = V_0 \cos\alpha \ , \forall t$$

$$V_x = \frac{dx}{dt} \Rightarrow \int dx = \int V_x dt$$

$$\Rightarrow x(t) = V_0 \cos\alpha \, . t + x_0$$

$$t = 0: \ V_x(0) = V_0 \cos\alpha \quad \& \ x_0 = 0.$$

$$\boldsymbol{x(t) = V_0 \cos\alpha . t}$$

ON oy :

$$a_y = \ddot{y} = -g \Rightarrow \frac{dV_y}{dt} = -g$$

$$\Rightarrow \int dV_y = \int -g dt \qquad \Rightarrow V_y(t) = -gt + C_2$$

t=o, $C_2 = V_y(0) = V_0 \sin\alpha$

$$\boldsymbol{\Rightarrow V_y(t) = -gt + V_0 \sin\alpha}$$

$$V_y = \frac{dy}{dt} \Rightarrow \int dy = \int V_y dt$$

$$\Rightarrow y(t) = -\frac{1}{2} g t^2 + V_0 \sin\alpha \ \ t + y_0$$

$$t = 0: \ y_0 = 0$$

$$\boldsymbol{y(t) = -\frac{1}{2} g t^2 + V_0 \sin\alpha . t}$$

The time equations of motion are:

- $\begin{cases} x(t) = V_0 \cos\alpha\, t \\ y(t) = -\frac{1}{2}gt^2 + V_0 \sin\alpha\, t \end{cases}$, $\vec{V} \begin{cases} V_x = V_0 \cos\alpha \\ V_y(t) = -gt + V_0 \sin\alpha \end{cases}$ & $\vec{a} \begin{cases} 0 \\ -g \end{cases}$

- <u>**The trajectory equation:**</u> is obtained by removing time : $\boldsymbol{y = f(x)}$
- $x(t) = V_0 \cos\alpha\, t \Rightarrow t = \frac{x}{V_0 \cos\alpha}$
- We substitute t in the equation for y(t):
- $y(t) = -\frac{1}{2}g\left(\frac{x}{V_0 \cos\alpha}\right)^2 + V_0 \sin\alpha\, \frac{x}{V_0 \cos\alpha}$
- $\boldsymbol{y(t) = -\frac{g}{2\, V_0{}^2 \cos^2\alpha} x^2 + \tan\alpha \,.\, x}$
- As: $y(t) = Ax^2 + Bx$: the equation of a parabola

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- **Maximum Height:** The object reaches its maximum height when its vertical velocity becomes zero.
- $\boldsymbol{h}: V_y(t_p) = 0,$ With t_p Peak time
- $V_y(t_p) = -g t_p + V_0 \, sin\alpha = 0 \Rightarrow t_p = \frac{V_0 \, sin\alpha}{g}$
- $\boldsymbol{h} = y(t_p) = -\frac{1}{2} g t_p^2 + V_0 \, sin\alpha \; t_p = -\frac{1}{2} g \left(\frac{V_0 \, sin\alpha}{g}\right)^2 + \frac{(V_0 \, sin\alpha)^2}{g}$
- $\boldsymbol{h} = \frac{\boldsymbol{(V_0 \, sin\alpha)^2}}{\boldsymbol{2g}}$
- The time at which the projectile reaches point I.
- $\boldsymbol{y(t_I) = 0} \Rightarrow -\frac{1}{2} g t_I^2 + V_0 \, sin\,\alpha \; t_I = 0$
- $\boldsymbol{t_I}\left(-\frac{1}{2} g \, t_I + V_0 \, sin\,\alpha\right) \boldsymbol{= 0} \Rightarrow \begin{cases} \boldsymbol{t_1 = 0} \quad \boldsymbol{(origine)} \\ \boldsymbol{t_I} = \frac{2 \, V_0 \, sin\alpha}{g} \end{cases} \Rightarrow \boldsymbol{t_I} = \frac{\boldsymbol{2 \, V_0 \, sin\alpha}}{\boldsymbol{g}}$

- **Range**: The range is the horizontal distance the object travels before hitting the ground.
- We plug $\mathbf{t_I}$ into x(t) : $x(\boldsymbol{t_I}) = V_0 \cos\alpha\, \boldsymbol{t_I} = V_0 \cos\alpha \frac{2\, V_0\, sin\alpha}{g}$
- $x(\boldsymbol{t_I}) = \frac{2V_0^2\, cos\ \alpha\ sin\alpha}{g}$
- $or : 2\ cos\ \alpha\ sin\alpha = sin2\alpha$, alors : $\boldsymbol{X_I} = \frac{V_0^2\ sin2\alpha}{g}$
- Calculation of the launch angle for which the range $\mathbf{X_I}$ is maximum :
- $\boldsymbol{X_I} = \frac{V_0^2\ sin2\alpha}{g}$ is max if : $sin\ 2\alpha = 1 \Rightarrow 2\alpha = \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{4}$

Chapitre IV : Dynamique
Chapter IV : Dynamics of particles

1. Introduction — **Introduction**
2. Systèmes étudiés et actions mécaniques — **Systems and mechanical actions**
3. Différents types de forces — **Types of forces**
4. Lois de Newton — **Newton’s Laws**

4.a. **1ère loi de Newton** (Principe d’inertie) — **1st Newton's law (Principle of inertia)**

4.b. **2ème loi de Newton** (Principe fondamental de la dynamique) — **2nd Newton's law** (Fundamental principle of dynamics)

4.c. **3ème loi de Newton** (Principe des actions réciproques) — **3rd Newton's law** (Principle of reciprocal actions)

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Chapitre IV : Dynamique
Chapter IV : Dynamics

5. Application (le pendule simple)	Applications (simple pendulum)
6. Moment d'une force	Moment of a force
7. Moment cinétique	Angular (Kinetic) Momentum
8. Théorème du moment cinétique (TMC)	Kinetic Momentum Theoreme (KMT)
9. Analogie entre grandeurs de translation et de rotation	Analogy between translation and rotation quantities

Introduction

- The kinematics of a point has allowed us to describe the motion of an object without considering the causes; it is dynamics that connects the motion to its causes.
- Newton established the fundamental laws of dynamics, notably the second law, which relates force and acceleration. In other words, it allows for connecting dynamic quantities (forces) to a kinematic quantity (acceleration).

Various types of forces

- There are two main categories of forces:
- **Forces of interaction at a distance:** for example, gravitational forces like weight, electromagnetic forces, and so on.......
- **Contact forces:** for example, frictional forces, tension in a string, reaction from a support, and so on......

poids d'un point matériel/ The weight of a point mass.

The weight of an object , with mass m, is primarily due to the gravitational attraction force exerted by the Earth on it. Weight refers to the force of gravity acting on an object with mass. Weight is the force with which an object is pulled toward the center of the Earth (or any other celestial body) due to gravity. It is a vector quantity, which means it has both magnitude and direction.

A point mass M with mass m is subject to its weight vector$\overrightarrow{W}$ or $\vec{\mathrm{p}}$, a vertical force directed downward, with a magnitude p =mg

$$\vec{P} = \mathrm{m}\,\vec{g}$$

The SI unit for weight is the newton (N), which is equivalent to one kilogram-meter per second squared (1 N = 1 kg·m/s²). m is the mass of the object in kilograms (kg).

g is the acceleration due to gravity, typically approximately 9.81 m/s² near the surface of the Earth.

forces de contact / Contact forces

- **Normal Force**: The force exerted by a surface to support the weight of an object resting on it. It acts perpendicular to the surface $\overrightarrow{R}$.
- **Frictional Force:** The force that opposes the relative motion or tendency of such motion of two surfaces in contact. It can be kinetic (when objects are sliding past each other) or static (when objects are at rest).
- **viscous drag or drag force:** when an object moves through a fluid (either a gas or a liquid). Viscous drag is a type of force that opposes the motion of an object through the fluid. This force is caused by the interaction between the object and the fluid, and it depends on several factors:
- **Spring Force:** The force exerted by a compressed or extended spring, which is proportional to the displacement from its equilibrium position.
- **Tension Force:** The force transmitted through a string, rope, cable, or any other type of flexible connector when it's pulled tight.

Force de Frottement/ Frictional Force

- **Frictional Force:** Friction opposes the relative motion or tendency of such motion of two surfaces in contact. It can be kinetic friction (opposing motion) or static friction (opposing impending motion).

- **kinetic friction :** also known as dynamic friction, is the force that opposes the relative motion or the tendency of such motion between two objects that are in contact and sliding past each other. It occurs when two surfaces are in motion with respect to each other and is generally characterized by the fact that the frictional force remains relatively constant as long as the objects are in motion. The magnitude of kinetic friction depends on the nature of the surfaces and the force pushing the objects together.

$$\boldsymbol{F_f} = \mu_{\boldsymbol{C}}\, \boldsymbol{R_n}$$

- **Static friction**: is the frictional force that prevents an object from initiating motion when a force is applied. It opposes the force trying to set an object in motion, keeping it at rest. The maximum static friction force is equal to the force applied, but it adjusts itself to match the applied force until the limit is reached. Once the applied force exceeds the maximum static friction, the object will begin to move, and kinetic friction takes over. Static friction is essential in keeping objects stable and preventing them from sliding or moving unintentionally.

$$\boldsymbol{F_f} = \mu_{\boldsymbol{S}}\, \boldsymbol{R_n}$$

Exemples de Coefficients de frottement statique et cinétique
Examples of static and kinetic friction coefficients

Matériaux	μ_S	μ_c
Acier sur glace	0,1	0,05
Acier sur acier	0,6	0,4
Bois Sur Bois	0,5	0,3
Teflon Sur Acier	0,04	0,04
Chaussure Sur glace	0,1	0,05
Pneu de voiture sur béton sec	1,0	0,7

lois de Newton / Newton's Laws

- The principles or laws are not demonstrated; it is from the observation of a large number of experiments that a physicist is led to formulate a law that will remain valid until another experiment challenges it.
- Classical mechanics is built upon three laws formulated by Newton.
- A material system is a collection of material points.
- A material system is isolated when there are no external actions exerted on the system, for example, an astronaut in space.
- A material system is pseudo-isolated when external actions acting on the system cancel each other out (it behaves as if it were isolated). For instance, on Earth, a system cannot be rigorously isolated because it inevitably experiences the influence of gravity.

vecteur quantité de mouvement
Vector momentum

The Vector momentum noted $\vec{P}$ of a materiel point whith mass m moving with a velocity vector $\vec{V}$ in a given reference frame is defined by:

$$\vec{P} = m\ \vec{V}$$

The SI unit of momentum is kilogram-meters per second (kg·m/s). This unit reflects that momentum is a product of mass and velocity.

$1^{ère}$ loi de Newton (Principe d'inertie)

1^{st} Newton's law (Principle of inertia)

First Law (Law of Inertia): An object at rest tends to stay at rest, and an object in motion tends to stay in motion with the same speed and in the same direction unless acted upon by an unbalanced external force. In other words, an object will not change its state of motion unless a force is applied to it.

« Dans un référentiel galiléen (R) , un système mécanique isolé ou pseudo-isolé est soit au repos soit en mouvement rectiligne uniforme. »

$$\vec{V} = C^{ste} \Rightarrow \vec{P} = m\ \vec{V} = C^{ste} \Rightarrow \frac{d\vec{P}}{dt} = 0\ ;\ V = C^{ste} = V_0 \text{ ou } V = 0 \text{ (objet au repos)}$$

This principle leads to the law of conservation of the total momentum of an isolated or pseudo-isolated system:

$$\vec{P} = \vec{P}' \quad \Rightarrow \quad m_1\vec{V}_1 = m_2\ \vec{V}_2 = m_3\ \vec{V}_3$$

From this first law arises the fundamental principle of statics:

$$\sum \vec{F} = \vec{0}$$

- If a system is in equilibrium, then $\overrightarrow{V}$ = o et $\vec{a}$ = o ⇒ $\sum \vec{F} = \vec{0}$

- However, the reverse is not true: if $\sum \vec{F} = \vec{0}$ ⇒ the system is either at rest or in uniform rectilinear motion ($\overrightarrow{V}$ = C^{ste}).

2ème loi de Newton/ Newton's Second Law (Principe fondamental de la dynamique) (Fundamental Principle of Dynamics)

As soon as a system is subjected to external forces, it is no longer isolated. The consequences include a change in motion, which is reflected in an alteration of the momentum vector, which is no longer conserved.

$$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} = \frac{d}{dt}(m\,\vec{V}_G) = m\frac{d\vec{V}_G}{dt} = m\vec{a}_G$$

$$\sum \vec{F}_{ext} = m\vec{a}_{\mathbf{G}}$$

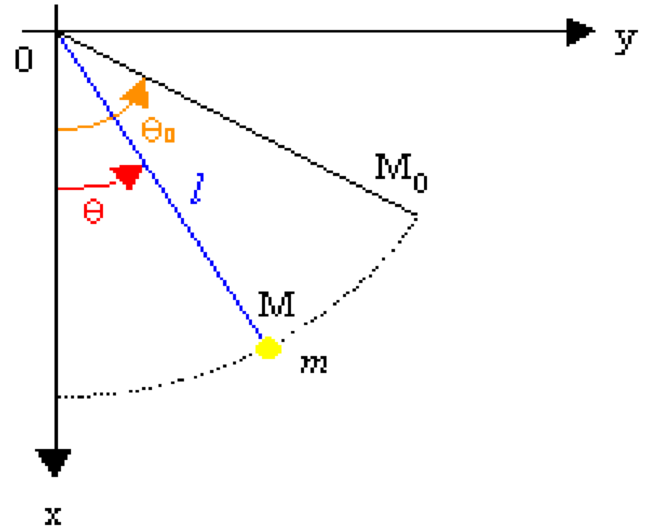
$3^{ème}$ loi de Newton/Newton's Third Law of Motion. (Principe des actions réciproques)

When two systems, S1 and S2, interact (either at a distance or through contact), each time system S1 exerts an action (a force) $\vec{F}_{1\text{-}2}$ on system S2, then system S2 exerts an action (a force) $\vec{F}_{2\text{-}1}$ on system S1.

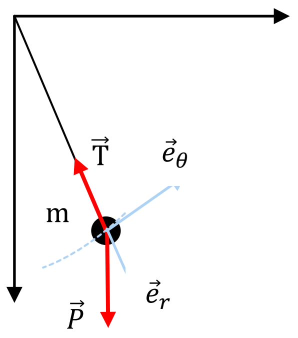
These forces are equal and opposite: $\vec{F}_{1\text{-}2} = -\vec{F}_{2\text{-}1}$

Application (le pendule simple)
Application (the simple pendulum)

A simple pendulum consists of a point mass m attached to the free end of a string with a length l. The mass is initially displaced from its equilibrium position by an angle θ_0, and then released with no initial velocity. Air friction is neglected.



To determine the equation of motion for a simple pendulum using the Fundamental Principle of Dynamics (Newton's Second Law), we can start by analyzing the forces acting on the mass m.



PFD : $\sum \vec{F}_{ext} = m\vec{a}_{G}$

By projecting onto the polar basis, we have:

$$mg\cos\theta - T = m\, a\, \vec{e}_r$$

$$-mg\sin\theta = m\, a\, \vec{e}_\theta$$

We know that in polar coordinates, the acceleration is given for circular motion(r = Cste)

$$\vec{a}(t) = -\ r\,\dot{\theta}^2\ \vec{e}_r + r\,\ddot{\theta}\ \vec{e}_\theta$$

With r = l , And by combining the two equations, we obtain:

$$\ddot{\theta} + \frac{g}{l}\sin\theta = 0$$

The pendulum is a harmonic oscillator if the angle θ is small enough so that $\sin(\theta) \approx \theta$, and the resulting differential equation can be linearized.

$$\ddot{\theta} + \frac{g}{l}\theta = 0$$

We define: $\sqrt{\frac{g}{l}} = W$ (pulsation)

$$\ddot{\theta} + W^2\theta = 0$$

This is the differential equation that describes the motion of a simple pendulum.

It's a second-order nonlinear differential equation that can be solved to find the equation of motion, which depends on the initial conditions (θ_0 and V_0).

The equation of motion for the pendulum is a second-order differential equation with no second member, and it has the following solution:

$$\theta(t) = A1 \cos wt + A2 \sin wt$$

The oscillations are sinusoidal (undamped harmonic oscillator) (oscillateur harmonique non amorti) because we have neglected air friction.

The constants A1 and A2 are integration constants that are determined from the initial conditions.

At: $t=0$, $\theta(t) = \theta_0 = A_1 \cos 0 + A_2 \sin 0 \Rightarrow A_1 = \theta_0$

At: $t=0$, $V(t) = V_0 = 0 = -A_1 \sin 0 + A_2 \cos 0 = 0 \Rightarrow A_2 = 0$

$$\theta(t) = \theta_0 \cos wt$$

The period is given by :

$$T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{l}{g}}$$

Moment d'une force
Torque or Moment of Force

- It is possible to express the Fundamental Principle of Dynamics in a different form by introducing an interesting new kinematic quantity when a system (point mass) rotates around a point or an axis :the moment of a force.
- the moment of a force $\vec{\mathrm{F}}$ with respect to a point O or an axis Δ expresses the force's ability to induce a rotation around point O or the axis Δ passing through O.
- The expression for the moment of a force is given by the vector cross product of the position vector $\overrightarrow{\mathrm{OM}}$ and the force $\vec{\mathrm{F}}$

$$\overrightarrow{\boldsymbol{M}}_{\mathbf{o}}(\vec{\boldsymbol{F}}) = \overrightarrow{\boldsymbol{OM}} \wedge \vec{\boldsymbol{F}}$$

$$\overrightarrow{M_o}(\vec{F}) = \overrightarrow{OM} \wedge \vec{F}$$

$$\overrightarrow{M_o}(\vec{F}) = OM.\,F.\,\sin\alpha\,\vec{\mu}$$

The unit of the moment of a force is the : N.m

A body is in equilibrium and at rest if:

1. The sum of the forces applied to it is zero.
2. The sum of the torques (moments) of the applied forces is zero.
3. The velocity of the body is zero.

Moment cinétique / Angular momentum

We call Angular momentum noted $\vec{L}_o$ ou $\vec{L}_\Delta$ of point M in rotation Around a point O or about the axis Δ passing through O, the moment of its angular momentum.

$$\vec{L}_o = \overrightarrow{OM} \wedge m\vec{V}$$

The unit of angular momentum is:$Kg.m^{-2}.S^{-1}$

In polar coordinates :

$$\vec{L}_o = r\vec{e}_r \wedge m(\dot{r}\ \vec{e}_r + r\dot{\theta}\ \vec{e}_\theta)$$

$$\vec{L}_o = r\vec{e}_r \wedge m\,\dot{r}\ \vec{e}_r + r\vec{e}_r \wedge m\,r\,\dot{\theta}\ \vec{e}_\theta$$

$$\vec{L}_o = m\,r^2\,\dot{\theta}\,\vec{K}$$

$$\vec{L}_o = J\,\vec{w}$$

The quantity $J = mr^2$ is called the moment of inertia

It describes the distribution of mass in space.

Théorème du moment cinétique (TMC)
theorem of angular momentum.

$$\frac{d}{dt}(\vec{L}_o) = \sum \vec{M}_0(\vec{F})$$

The proof of the theorem of angular momentum can be summarized as follows:

$$\vec{L}_o = \overrightarrow{OM} \wedge m\vec{V}$$

We differentiate this expression with respect to time:

$$\frac{d}{dt}(\vec{L}_o) = \frac{d}{dt}(\overrightarrow{OM} \wedge m\vec{V})$$

$$\frac{d}{dt}(\vec{L}_o) = \frac{d}{dt}\overrightarrow{OM} \wedge m\vec{V} + \overrightarrow{OM} \wedge \frac{d}{dt}m\vec{V}$$

$$\frac{d}{dt}(\vec{L}_o) = \overrightarrow{OM} \wedge m\vec{a} = \overrightarrow{OM} \wedge \sum \vec{F}_{ext} = \sum \vec{M}_0(\vec{F})$$

$$\frac{d}{dt}(\vec{L}_o) = \sum \vec{M}_0(\vec{F})$$

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Analogie entre grandeurs de translation et de rotation
Analogy between translational and rotational quantities

Vitesse linéaire/velocity	$\vec{V}$	Vitesse angulaire/angular velocity	$w = \dot{\theta}$
Accélération/acceleration	$\vec{a}$	Accélérationangulaire/ angular acceleration	$\ddot{\theta} = \dot{\omega}$
Force/force	$\vec{F}$	Moment de force/Torque	$\vec{Mo}\,(\vec{F})$
Masse(inertie)/ mass	M	Moment d'inertie/Moment of Inertia	$J = mr^2$
Quantité de mouvement momentum	$\vec{P} = m\,\vec{V}$	Moment cinétique/Angular momentum	$\vec{Lo} = J\,\vec{w}$
Energie Cinétique//Kinetic Energy	$Ec = \frac{1}{2} m V^2$	Energie Cinétique/Kinetic Energy	$Ec = \frac{1}{2} J w^2$

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