TP-3
SIMPLE PENDULUM

The Purpose

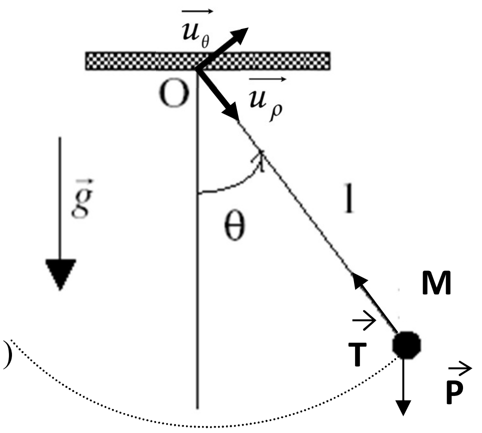
1/ Study the period (***T***) of a simple pendulum: influence of the length (***L***) of the pendulum and the amplitude (angle ***θ***) on the period of oscillations.

2/ Measurement of the intensity of the gravitational field using a simple pendulum.

Theoretical part

We consider a simple pendulum consisting of a material point (***M***) with mass ***m***, suspended from an inextensible, massless string of length ***l***.

This pendulum oscillates without friction (neglecting air resistance) in a vertical plane (figure opposite).



The mass m is displaced by an angle θ_0 with respect to the vertical. The second law of Newton in a Galilean reference frame is written as:

$m\vec{\gamma} = \vec{P} + \vec{T}$ ....(1)

$\vec{\gamma}$=?

In the polar coordinate system, we have:

$\vec{v} = \frac{d\vec{r}}{dt}$ / $\vec{r} = r\vec{u}_\rho$

$\vec{v} = \frac{d}{dt}(r\vec{u}_\rho) = \dot{r}\vec{u}_\rho + r\dot{\theta}\vec{u}_\theta$ ....(2)

$\vec{\gamma} = \frac{d^2}{dt^2}(\vec{r}) = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\dot{r}\vec{u}_\rho + r\dot{\theta}\vec{u}_\theta)$

$\vec{\gamma} = (\ddot{r} - r\dot{\theta}^2)\vec{u}_\rho + (2\dot{r}\dot{\theta} + r\ddot{\theta})\vec{u}_\theta$ ....(3)

According to (1) and (3) :

$m[(\ddot{r} - r\dot{\theta}^2)\vec{u}_\rho + (2\dot{r}\dot{\theta} + r\ddot{\theta})\vec{u}_\theta] = (-T + P\cos\theta)\vec{u}_\rho - P\sin\theta.\vec{u}_\theta$ ..... (4)

With $r = L = cst \Leftrightarrow \ddot{r} = \dot{r} = 0$

So: $(4) \Leftrightarrow \begin{cases} mr\dot{\theta}^2 = -T + P\cos\theta & \text{(5 - a)} \\ mr\ddot{\theta} = -P\sin\theta & \text{(5 - b)} \end{cases}$

For weak oscillations (θ≤10°): $\sin\theta = \theta$ (rad).

so:

$(5-b) \Leftrightarrow mr\ddot{\theta} + mg\theta = 0 \Leftrightarrow L\ddot{\theta} + g\theta = 0 \Leftrightarrow \ddot{\theta} + \frac{g}{L}\theta = 0$ on pose $w^2 = \frac{g}{L}$ we will have the differential equation: $\ddot{\theta} + w^2\theta = 0$ (6)

Whose general solution is: $\theta(t) = \theta_1\cos\omega t + \theta_2\sin\omega t$

For $\theta(t=0) = \theta_0 \Leftrightarrow \theta_1 = \theta_0$ with $v(t=0) = 0 \Leftrightarrow [r\dot{\theta}(t)]_{t=0} = 0$

$\Leftrightarrow r[-\theta_1\sin wt + \theta_2\cos wt]_{t=0}$ that is $\theta_2 = 0$

Therefore $\theta(t)=\theta_0\cos wt \Leftrightarrow \theta(t)=\theta_0\cos\frac{g}{L}t$ ....(7)

This is the case of harmonic oscillatory motion, with its period given by:

$$T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{L}{g}} \quad \text{..........(8)}$$

Experiment

I/ Influence of the pendulum's length on the period

Attach a mass m to the pendulum ($m = 33g$).

Oscillate the pendulum with a fixed angle θ_0 ($\theta_0 \leq 10°$), measure the time ($t = nT$), then calculate the period for various lengths of the string and for two numbers of oscillations ($n_1=10$ et $n_2=30$).

Record the results in **Table 1**.

$\theta_0=7°, n_1=10, n_2=30$				
L (cm)	20	40	70	100
$t_1=n_1T_1(s)$				
$T_1(s)$				
$T_1^2\ (s^2)$				

Table 1

1/ Plot the graph $L\left(\frac{T^2}{4\pi^2}\right)$ and determine the average value of **g** and calculate its uncertainty with δt **= 0.02s** and δl **= 0.001m**.

2/ Compare the results of the period T and the value of g for n_1 and n_2. What can you conclude ?

II/ Influence of the angle on the period

Attach a mass m to the pendulum at a length L ($L = 40$ cm).

Spread the pendulum with different angles θ_0 and release it without initial speed and measure the time t and calculate the period of the oscillations (for n_1 and n_2 oscillations).

Record the results in Table 2.

L = 40 cm , $n_1=10$, $n_2=30$					
θ_0(°)	6	8	10	30	90
$t_1=n_1T_1(s)$					
$T_1(s)$					

Table 2

1/ Calculate g analytically for the angles that allow the application of the formula $T=2\pi\sqrt{\frac{L}{g}}$, and calculate its uncertainty.

2/ Does the angle variation θ_0° influence T and g? Interpret the results.

- Conclusion