جامعـــــة باتنـــــة 2
الشهيد مصطفى بن بولعيد

Chemistry 1
Structure of matter

Commun core in sciences and technology

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Chapter 1:

Components of Matter

Chapter 1: Components of Matter

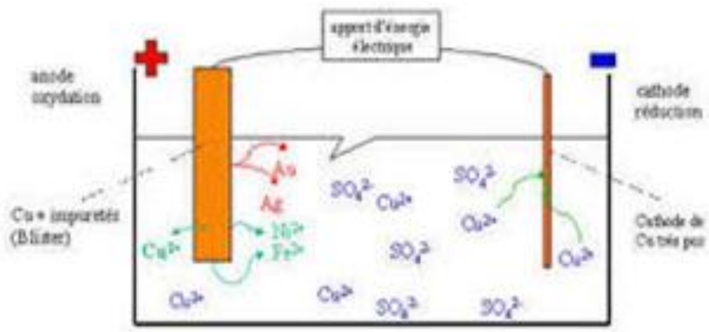
1. Introduction:

In this chapter, we will take a look at numerous experiments that contributed to the development of the atomic structural theory, and how they helped in the evolution of this theory. It's characterized by three fundamental stages:

1. Discovery of electricity and the electrical nature of matter in 1900.
2. Discovery that the atom consists of a nucleus and an electron cloud in 1911.
3. Discovery of the mechanical laws governing the behavior of electrons within the atom in 1925.

2. The Electron:

1. Electrical Nature of Matter: One of the most significant early indications in discovering the electrical nature of matter and the relationship between matter and electricity emerged from the experimental research conducted by scientist Faraday in 1833 in the field of electrochemical analysis.



Device Components:

- Positive Electrode = Anode+
- Negative Electrode = Cathode–

When passing an electric current through this solution consisting of dissolved copper sulfate in water:

- Positive ions (Cu++) move towards the negative electrode.
- Negative ions ($SO4^{-2}$) move towards the positive electrode.

Result:

- It has been found that the weight of the substance deposited on one of the electrodes is proportional to the amount of electricity passing through the solution.
- The product of the weights of the deposited, emitted, or dissolved substances on these electrodes in the quantity of the resulting electricity is proportional to the equivalent weights of the substances.
- If we represent q as the amount of electricity associated with the appearance of a copper atom at the negative electrode.
- Therefore, 2q, 3q, ..., nq represents the amount of electricity resulting from the deposition of 2, 3, ..., n copper atoms at the cathode.
  - The emergence of these whole number quantities of electricity led scientists to assume that electricity is composed of elementary charges, and that atoms contain such charges.

2. Proof of the Existence of the Electron:

The decisive experimental evidence for the existence of electrons and the determination of their properties came after the year 1897 through research conducted on the electrical conduction of gases under low pressure, the "Croukes Experiment".

Gases are usually insulators of electricity. However, when exposed to high voltage differentials (Haute tension) and under low pressure of less than 0.01 atmospheres, their resistance collapses, allowing the passage of electric current through them, and this is accompanied by light emission.

- When reducing the pressure to 4-10 atmospheres, electrical conductivity continues, but the gas glow weakens.
- If the voltage difference is very high, ranging from 5000 to 10000 volts, the inner wall of the glass tube (the side opposite to the cathode) becomes luminous.
- Scientists explained this glow as a result of bombarding the inner wall of the glass with radiation emitted from the cathode. Hence, it was named "cathode rays," and it is characterized by the following properties:
  1. Emit from the cathode, hence the name "cathode rays."
  2. Travel in straight lines: Experimenting with a metal barrier in their path and observing their silhouette on the glass tube wall (Figure 2).
  3. Deflect from their straight path when exposed to an electric or magnetic field.
  4. Carry a negative charge.

5. Possess kinetic energy: This was proven by the rotating wheel experiment moving toward the anode (Figure 3).

$$E_c = \frac{Nmv^2}{2}$$

These elementary charges were called "electrons" by Stoney.

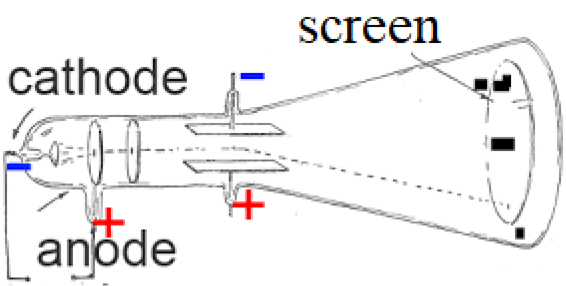


Figure 1

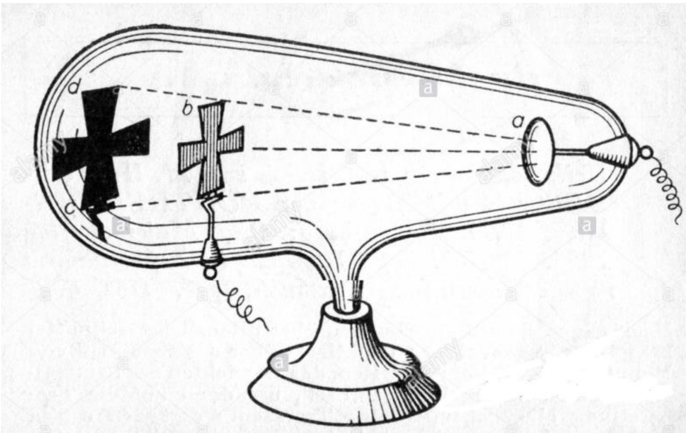


Figure 2

Figure 3

3. Measurement of the Relative e/m Ratio (J.J. Thomson's Experiment):

The measurement of the relative e/m ratio is based on the measurement of the deflection of a particle under an electric or magnetic field. Thomson proved that the e/m ratio is independent of the experimental conditions, where: e/m = -1.7588×10^11 $C.kg^{-1}$

The experiment is as follows:

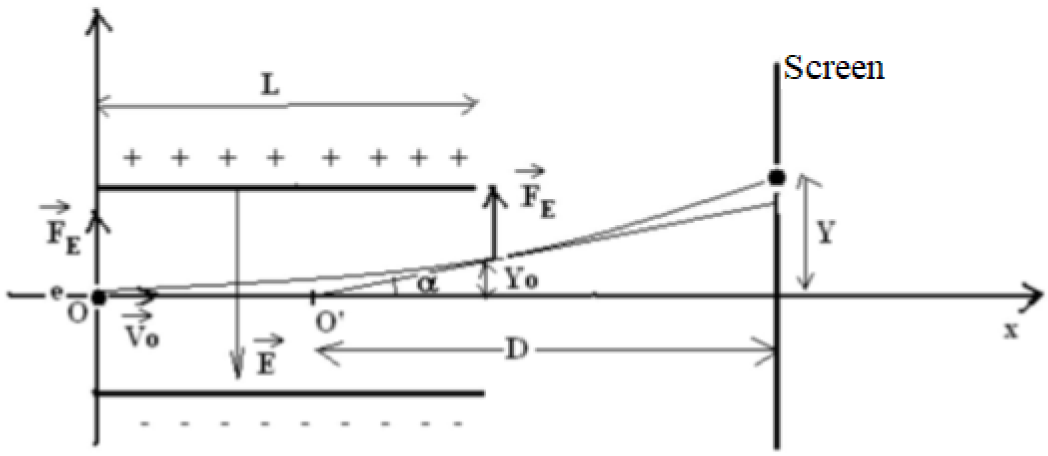
- Cathode ray radiation is exposed to a uniform electric field E (between plates of length l).
- The objective is to deduce the value of the deflection Ys that a bundle of electrons undergoes upon exiting the capacitor, where their mass is m and their charge is -e.

Note that their initial velocity is perpendicular to the intensity of the electric field.

- We measure the deflection Ys and subject it to the effect of a magnetic field B acting over the same distance (length of the capacitor).
- This electron is subject to the Coulomb force, **q = -e**, where **F = q.E = -e.E**.
- Since the weight of the electron is negligible, **F' = m $_{e-}$. g = 0.**

According to basic principles of dynamics:

Along the x-axis:



According to Newton's law

$$F = m_e \gamma_x = m_e \frac{d^2x}{dt^2} = 0 \Rightarrow \frac{dx}{dt} = v_0$$

$$dx = v_0 dt \Rightarrow \int dx = \int v_0 dt \Rightarrow x = v_0.t \quad \dots\dots(1)$$

Along the y-axis:

$$F = m_e \gamma_y = m_e \frac{d^2y}{dt^2} = e.E \Rightarrow \frac{d^2y}{dt^2} = \frac{eE}{me}$$

$$\int \frac{d^2y}{dt^2} = \int \frac{eE}{m_e} \Rightarrow \frac{dy}{dt} = \frac{eE}{me}t \qquad = v_y$$

$$dy = \frac{eE}{me}tdt \Rightarrow \int dy = \frac{eE}{m_e}\int t.dt$$

$$\Rightarrow y = \frac{1}{2}\frac{eE}{m_e}t^2 \ldots\ldots\ldots(2)$$

Substituting in the value of t from relation 1:

$$y_s = \frac{1}{2}\frac{Ee}{me}(\frac{x}{v_0})^2 = \frac{1}{2}\frac{eE}{m_e}\frac{x^2}{v_0^2}$$

$x = l$

$$y_s = \frac{1}{2}\frac{e.E}{m_e}\frac{l^2}{v_0^2}$$

This is the deviation that the electron undergoes upon exiting the electric field.

When applying the magnetic field, a force arises that deflects the electron downward, known as the Laplace force.

$$F' = qv_0B$$

In order for the magnetic force F' to be equal to the electric force F it must be

$$F' = -F = -(-eE)$$

$$qv_0B = +eE$$

$$qv_0B = +eE \Rightarrow v_0$$

compensation in the previous relationship

$$y_s = \frac{1}{2}\frac{eE}{me}\frac{l^2}{v_0^2} = \frac{1}{2}\frac{eE}{m_e}\frac{l^2}{\frac{E^2}{B^2}}$$

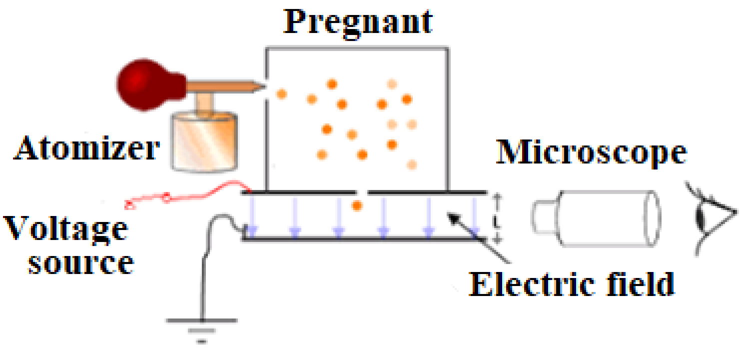
$$\Rightarrow y_s = \frac{1}{2}\frac{el^2B^2}{m_eE^2}$$

$$\Rightarrow \frac{e}{m_e} = \frac{2y_sE}{l^2B^2}$$

4. Determining the Charge of the Electron: Millikan Experiment

The apparatus used is illustrated in the following diagram and consists of the following components:

1. A chamber containing air confined between charged plates of a capacitor.
2. An X-ray source (RX) on one side of the chamber.
3. One of the capacitor plates (the upper one) is perforated to allow oil droplets to pass into the ionization chamber.
4. A spray for emitting small oil droplets.
5. A microscope to observe the falling droplets and monitor their speed.



1. In the absence of an electric field E:

The droplet is subjected to three forces:

a. **The gravitational force**, **p=mg**, where:

m: is the mass of the droplet.

g: is the gravitational acceleration.

ρ: is the volume mass density of the droplet.

V: is the volume of the droplet.

$$\mathrm{p} = \rho.\mathrm{V}.\mathrm{g}$$

And since the droplet has a spherical shape, its shape does not change during movement.

$$P = \rho \frac{4}{3}\pi r^3 g$$

$$P = \frac{4}{3}\pi r^3 \rho g$$

Where

r: is the radius of the sphere.

b: The air resistance force

$$F_f = -6\pi n r v_0$$

Where:

V_0 : Droplet velocity

n : Air viscosity coefficient

c- The Archimedean buoyant force:

$$\pi_A = -V\rho_a g = -\frac{4}{3}\pi r^3 \rho_a g$$

Where:

ρ_a : Volume mass density of air

The sum of these forces = Coulomb force

$$m\gamma = F + F_f + \pi_A$$

$$m\gamma = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho_a g - 6\pi n r v_0$$

$$m\gamma = \frac{4}{3}\pi r^3 (\rho - \rho_a) g - 6\pi n r v_0$$

The velocity becomes constant when $\gamma = 0$, and we denote it as v_1, which means acceleration has become zero.

$$\Rightarrow 6\pi n r v_1 = \frac{4}{3}\pi r^3 (\rho - \rho_a) g$$

$$v_1 = \frac{2r^2(\rho - \rho_a)g}{9n}$$

We observe that $v_1 < 0$ because $\rho_a > \rho$.

To calculate the time it takes for the droplet to descend a distance d, you can use the observation with the microscope: $|v| = \frac{d}{t}$.

2- In the presence of the electric field E:

In this case, we observe the upward and downward movements of the droplets because they become charged due to collisions with air molecules ionized by X-rays. Consequently, each droplet is exposed, in addition to the previous three forces, to the electric Coulomb force.

$$F_{el} = qE$$

The previous equation can be written as follows:

$$m\gamma = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho_a g - 6\pi n r v_0 + qE$$

$$= \frac{4}{3}\pi r^3 (\rho - \rho_a) g - 6\pi n r v_0 + qE$$

The limit velocity becomes when $\gamma = 0$, and we denote it as v_2.

$$6\pi n r v_2 = \frac{4}{3}\pi r^3 (\rho - \rho_a) g + qE$$

$$v_2 = \frac{2r^2(\rho - \rho_a)g}{9n} + \frac{qE}{6\pi n r}$$

$$v_2 = v_1 + \frac{qE}{6\pi n r}$$

$$q = \frac{6\pi n r}{E}(v_2 - v_1)$$

The result is that the calculated value q for each droplet does not change except in multiples of the elementary charge, which is the charge of an electron (e).

The charge of an electron is the fundamental unit of electric charge.

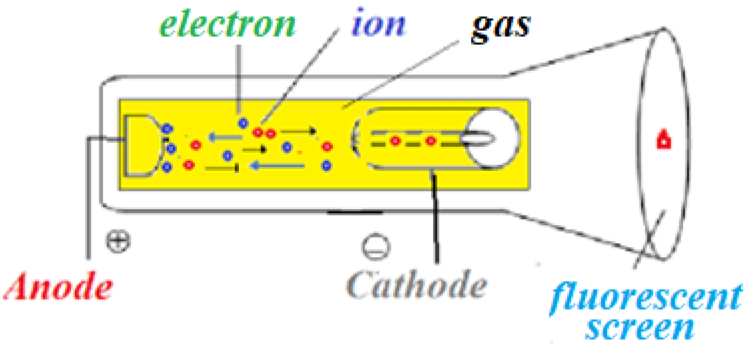
$$q = e = -1{,}6 \times 10^{-19} coulomb$$

$$m_e = 9{,}109 \times 10^{-31} kg$$

5. The atomic nucleus:

A. X-rays: Goldstein's Experiment

In this experiment, the scientist Goldstein used the same apparatus that Crouks used, with one key difference, which is the perforated cathode, as shown in the figure:



1.When a high voltage difference is applied and the pressure is reduced to 0.01 atmospheres or less, gas molecules lose their electrons.

2. These resulting positive ions are attracted by the cathode and gain sufficient kinetic energy to penetrate the channels in the perforated cathode, colliding with the fluorescent screen.

3. These ions are called X-rays.

The result:

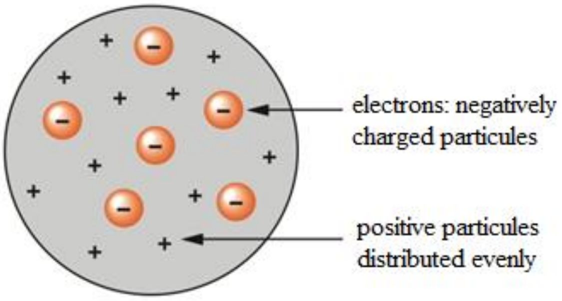
1. These X-rays scatter in the opposite direction of the cathode rays.
2. They deflect under the influence of both electric field E and magnetic field B in a direction opposite to the deflection of the cathode rays.
3. Consequently, they carry a positive charge.
4. Their deflection is at a smaller angle than the deflection of cathode rays because they are heavier.

And since the charge and mass of the electron are known, and considering the small mass of the electron (me-), scientists assumed that the atom is mostly composed of positively charged mass. Therefore, this mass makes up most of the atom's volume. Until Thomson came along, proposing that the atom, in its smallness, is:

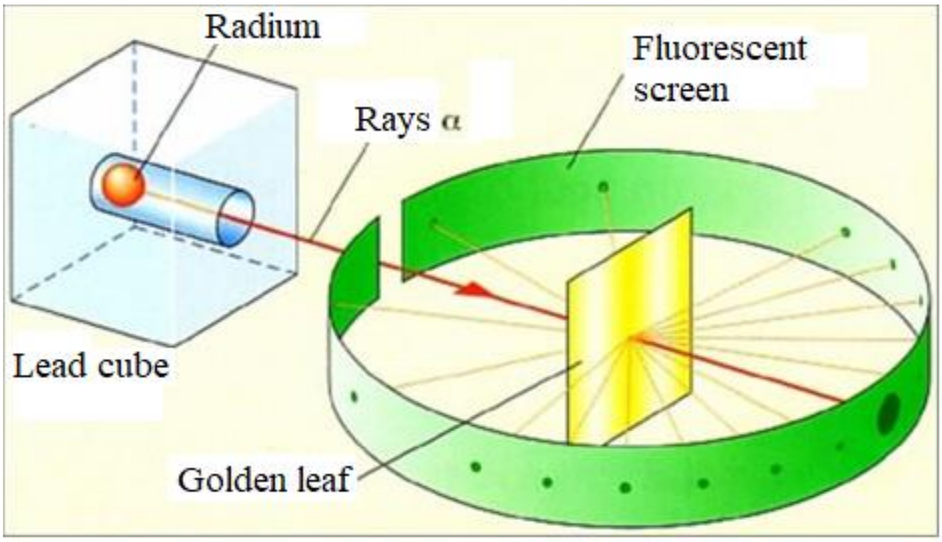
- A sphere with a radius R on the order of 10^{-8} cm.
- This sphere contains two types of electricity:
    A. Uniformly charged with a positive charge.
    B. Electrons with a negative charge that vibrate within this sphere.

- The charge of all electrons is equal and opposite in sign to the total positive charge of electricity.
- This gives the atom its neutral charge, as shown in the diagram.

B. Discovery of the Nucleus: Rutherford's Experiment 1911



Thomson's previous proposal was challenged after Rutherford published his results in 1911. The following figure illustrates the experimental setup:



Experiments by Rutherford greatly contributed to the development of atomic structure theory. In this experiment, Rutherford studied the deflection of alpha particles $({}^{4}_{2}He^{++}) \equiv$ (α), by directing them at a thin metal foil (gold foil).

The experiment:

Alpha particles $({}^{4}_{2}He^{+} \equiv \alpha^{+})$with known initial velocities were sent in a tightly focused beam, obtained using lead barriers.

This beam collided with a thin metal foil made of gold.

The deflection of the alpha particles was monitored either by a specialized counter or by the light emissions resulting from the collision of α particles with a screen coated with zinc sulfide (ZnS).

Observations:

1. Most alpha particles passed through the gold foil without any significant deflection (θˆ = 0°).
2. Approximately one out of every 100 alpha particles was deflected at angles between 0° and 45°.
- About one out of every 10,000 alpha particles experienced deflections ranging from 45° to 150°.
3. One out of every 10^8 alpha particles underwent complete reflection, meaning they were deflected by an angle of θˆ= 180°. Knowing that the collision of (α) with the electrons does not cause any deviation due to the small mass of the electrons compared to the mass of the (α) rays, $m_\propto=8000m_e$.

The Result (Rutherford's Model):

The experiment demonstrated that the majority of the atom's mass is concentrated in small positively charged centers called nuclei. Electrons orbit around these nuclei in specific orbits. It was a groundbreaking finding that replaced Thomson's model and paved the way for our modern understanding of atomic structure.

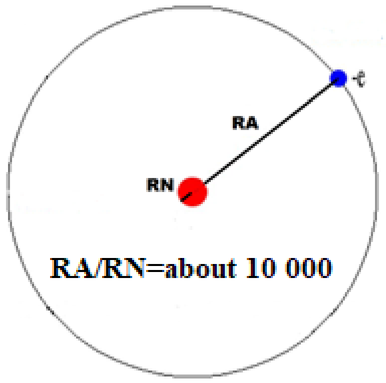
$$\textbf{Ratio calculation: } \frac{R_N}{\mathrm{R_A}} == \frac{\textbf{nucleus's radius}}{\textbf{atom's radius}}$$

It was found that the radius of the nucleus $RN=10^{-14}$m and the radius of the atom of the order $RA=10^{-10}$m.

$$\frac{R_N}{R_A} = \frac{10^{-14}}{10^{-10}} \Rightarrow \frac{R_N}{R_A} = 10^{-4} \Rightarrow \frac{R_N}{R_A} = \frac{1}{10^4}$$

$$\Rightarrow \mathrm{R_A} = 10^4\mathrm{R_N}$$

If we look at this ratio, we find that the dimensions of the nucleus and alpha particles (α) are very widely spaced apart. Therefore, the likelihood of alpha particles encountering the nucleus of gold is extremely small.



6.Components of the nucleus:

1. Protons: Protons were discovered by Goldstein in the experiment with alpha rays, by creating a high voltage difference in a tube filled with hydrogen gas (H2) at low pressure (0.01 atm).

- **Accelerated electrons under high voltage**: These accelerated electrons collide with H_2 molecules, ionizing them to produce H+ ions, which are protons (p=H+).
- These protons are observed after passing through the perforated cathode.
- Analyses have shown that:

$$q=+e=+1,6022 \times 10^{-19} \text{ coulomb}$$
$$m_p=1,6726 \times 10^{-27} kg$$

2. Neutrons: Neutrons were discovered by Chadwick in 1932 using his bombardment of beryllium (Be), lithium (Li), and boron (B) with alpha rays (He++≡α42), which are helium nuclei.

When beryllium is bombarded with alpha rays (α), it produces various types of particles that, when they collide with paraffin rich in hydrogen, emit protons, as in the equation:

$$^{9}_{4}Be + {}^{4}_{2}He \rightarrow {}^{1}_{0}n + {}^{12}_{6}C$$

- When an electric field is applied in the path of these particles, they do not deflect.
- Similarly, they do not deflect under the influence of a magnetic field.
- Therefore, they are uncharged radiation.
- They have a high penetrating power.

- And Chadwick named them neutrons.

$$q=0$$
$$m_n=1{,}6749 \times 10^{-27}kg$$

You can represent an atom symbolically as follows:

Let's assume an atom with an atomic number Z, which is typically electrically neutral. If it has Z electrons, its nucleus contains Z protons. The mass of the atom mainly comes from the protons and neutrons, as the mass of electrons is practically negligible. If we symbolize the number of neutrons in the nucleus as N and the number of protons as Z, then we call A the mass number, which equals the number of protons + the number of neutrons: A = Z + N.

The atom can be represented as follows:

$$^{A}_{Z}X \equiv {}^{16}_{8}O$$

$A=Z+N=16 \quad Z=8 \Longrightarrow Z+N=16 \Longrightarrow N=16-8=8$

This atom contains:

Z=8 electrons

Z=8 protons

N=8 neutrons

Isotopes:

Atoms or nuclides that have the same atomic number but different mass numbers are isotopes of the same element. They differ only in the number of neutrons. Example 1: The Three Hydrogen Isotopes:

$$^{1}_{1}H \begin{cases} A=1 \\ Z=1 \\ N=0 \end{cases} \qquad ^{2}_{1}H \begin{cases} A=2 \\ Z=1 \\ N=1 \end{cases} \qquad ^{3}_{1}H \begin{cases} A=3 \\ Z=1 \\ N=2 \end{cases}$$

Example 2: The three isotopes of carbon:

$$^{12}_{6}C \begin{cases} A=12 \\ Z=6 \\ N=6 \end{cases} \qquad ^{13}_{6}C \begin{cases} A=13 \\ Z=6 \\ N=7 \end{cases} \qquad ^{14}_{6}C \begin{cases} A=14 \\ Z=6 \\ N=8 \end{cases}$$

- Isotopes have similar physical properties.
- They also have similar chemical properties because they have the same Z.
- The difference in isotopes is in the mass of the nucleus because its N is different.

Isobares: are nuclides that have the same mass number but differ in the number of protons and neutrons.

Example:

- $^{33}_{16}S, ^{33}_{17}Cl$ have the same mass number (A=33).
- $^{35}_{15}P, ^{35}_{16}S, ^{35}_{17}Cl$ have the same mass number (A=35).

Isotons: are nuclides that have the same number of neutrons, but differ in the number of their protons and therefore in their mass number.

Example:

- $^{33}_{16}S, ^{34}_{17}Cl, ^{35}_{18}Ar$ have the same number of neutrons (n=17).
- $^{35}_{15}P, ^{37}_{17}Cl$ have the same number of neutrons (n=20).

Atomic Molar Mass of an Element:

In its natural state, elements are often composed of a mixture of isotopes with constant ratios.

A. The isotopic abundance (x_i) is the mass percentage of isotope i in the natural element, defined as follows:

$$\boxed{\sum x_i = 1}$$

Example 1: Natural hydrogen: It consists, as we know, of three isotopes, as shown in the figure:

الهيدروجين	^1_1H	^2_1H	^3_1H
الوفرة x_i	99,985%	0,015%	10^{-4}%≤

$$\sum Xi = 0{,}99985 + 0{,}015 = 1{,}01485$$

$$\boxed{\Rightarrow \sum x_i = 1}$$

B. Definition of relative isotopic mass:

It is symbolized by the symbol Mi, and by definition it is:

$$\mathrm{M(i)} = \frac{mass\ of\ isotope(i)}{\frac{1}{12}\text{mass of atom of}\,{}^{12}_{6}\mathrm{c}}$$

- Its unit is the atomic mass unit (u.m.a) in the case of a single atom.
- In the case of a mole of atoms, its unit is (g), and the relative isotopic mass calculated in practice is always close to the mass number A, meaning that:

$$\boxed{\mathrm{A} \approx Mi}$$

Example : Natural oxygen consists of three isotopes, as well as carbon, as in the following table:

Natural oxygen	${}^{16}_{8}\mathrm{O}$	${}^{17}_{8}O$	${}^{18}_{8}\mathrm{O}$
Abundance x_i	99,76%	0,04%	0,20%
relative isotopic Mi mass	15,9950	16,9992	17,9993
Natural carbon	${}^{12}_{6}c$	${}^{13}_{6}c$	${}^{14}_{6}c$
x_i	98,89%	1,11%	$\leq 10^{-4}$
M_i	12	13,0063	14,0032

C. Atomic molar mass of an element: M

It is the sum of the product of the natural abundances of each isotope by its relative isotopic mass, and its unit is: g/mol.

$$\boxed{\mathrm{M} = \sum x_i M_i}$$

Example 1: Calculate M for oxygen, which is equal to 15.9994 g/mole.

$$M = x_{16}M_{16} + x_{17}M_{17} + x_{18}M_{18}$$

$$= 0{,}9976 \times 15{,}9950 + 0{,}0004 \times 16{,}9992 + 0{,}0020 \times 17{,}9993$$

$$= 15{,}956612 + 6{,}79968.10^{-3} + 0{,}0359986$$

$$\mathrm{M} = 15\,.99941\,{}^{g}/_{mole}$$

Example 2: Calculate M using the previous table:

$$M = x_{12}M_{12} + x_{13}M_{13} + x_{14}M_{14}$$

$$M = 0{,}9889 \times 12 + 0{,}0111 \times 13{,}0063 + 0 \times 14{,}003$$

$$\mathrm{M} = 11{,}8668 + 0{,}1443 + 0$$

$$\mathrm{M} = 12\,.0111\, {}^{g}/_{mole}$$

We note that it is always close to the mass number of the most abundant isotope in nature, but if there is no isotope present in greater abundance than the others, then the atomic molar mass of the element is an integer number away from the mass number.

For example, in the case of chlorine:

Natural chlorine	${}^{35}_{17}Cl$	${}^{37}_{17}\mathrm{Cl}$
x_i	75,8%	24,2%
M_i	34,97	36,97

$$M = x_{35}M_{35} + x_{37}M_{37}$$

$$= 0{,}758 \times 34{,}97 + 0{,}242 \times 36{,}97$$

$$= 26{,}50726 + 8{,}94674$$

$$\mathrm{M} = 35{,}454\, {}^{g}/_{mole}$$

We note that the difference is clear in this case, estimated at a full number in the case of ${}^{37}_{17}Cl$ and 1/2 a number in the case of ${}^{35}_{17}Cl.$

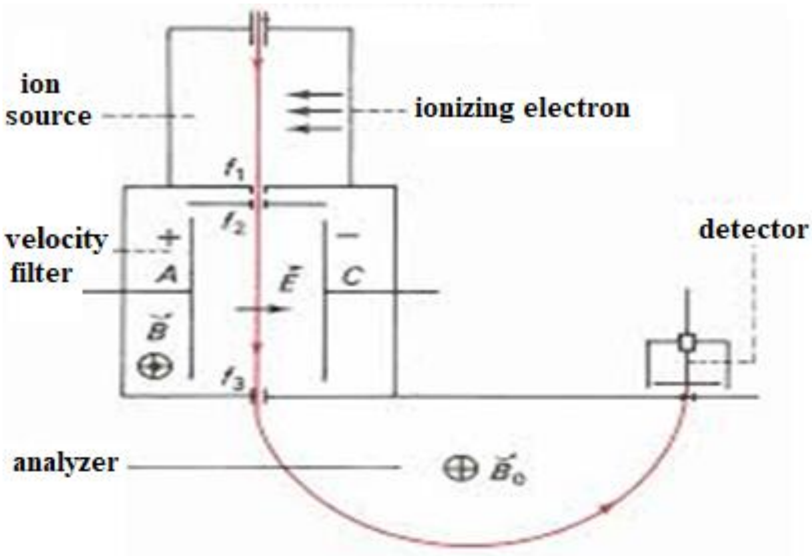
D. Isotope Separation:

Isotope separation necessarily involves the use of physical methods because isotopes share the same chemical properties due to having the same number of electrons. This separation relies on their differences in mass and utilizes devices called mass spectrometers.

There are two types of mass spectrometers:

1. Mass spectrometers that use ions or particles with different velocities (e.g., Thomson, Aston, Goldsmith).
2. Mass spectrometers that use ions or particles with the same velocity (e.g., Bainbridge, Dempster).

In our study, we will focus on the mass spectrometer known as Bainbridge.



Schematic diagram of the Bainbridge spectrometer

The apparatus shown in the figure consists of the following components:

A. Ion Source :

The ion source can be solid, liquid, or gaseous.

- A filament and wire are used, with the desired salt or metal placed on the wire.
- The wire is heated by passing an electric current through it.
- This causes the emission of electrons from the wire, which ionizes the metal or salt to produce positively charged ions.
- Some of these ions are ejected through openings f1 and f2 and are accelerated in the ionization chamber using a weak electric field E'.
- In the case of a gas, an electron gun is used, as shown in the figure.
- The gas is bombarded with electrons from the electron gun, leading to the ionization of the gas into both positively and negatively charged ions. The negatively charged ions are filtered out using a lift in the ionization chamber.

B. Ionization Chamber (Chambre d'ionisation):

In this chamber, the positive ions are accelerated by a voltage difference (ddp) and enter the velocity filter through the aperture f2.

C. Velocity Filter:

The ions enter the velocity filter with different velocities, and the filter is based on the velocity-dependent effect of two perpendicular fields, an electric field (E) and a magnetic field (B) (E⊥B), and both fields are perpendicular to the ion flow direction.

- The apertures f1, f2, and f3 are aligned in a straight line. Only the ions that experience a deflection due to the electric field E equal in magnitude but opposite in direction to the deflection they experienced under the influence of the magnetic field B will pass through f3 and proceed in a straight path to the analyzer.

That is:

$$|f_m|=|f_{el}|$$

Electric force = magnetic force

$$qvB = qE \Rightarrow v = \frac{E}{B}$$

When the ions leave the velocity analyzer and enter the analyzer, their velocity is $v = \frac{E}{B}$

D - Analyzer:

The ions that enter the analyzer form a homokinetic beam, and this beam of ions is subjected to a new perpendicular magnetic field B_0 at every moment along their path. The ions experience a force perpendicular to the tangent of their path due to the law of Laplace force:: $f = qvB_0$.
As a result, the ions exhibit regular circular motion with acceleration:

$$\gamma = \frac{v^2}{R}$$

$$f = m\gamma = m.\frac{v^2}{R}$$

$$f = qvB_0$$

$$\Rightarrow m.\frac{v^2}{R} = qvB_0 \Rightarrow m.\frac{v}{R} = qB_0$$

Since: $:v = \frac{E}{B}$, then:

$$\frac{m}{R}\frac{E}{B} = qB_0 \Rightarrow \frac{m}{q} = \frac{B_0BR}{E}$$

$$\Rightarrow {}^{q}/_{m} = \frac{E}{(B_0 B)R}$$

Since: constant $=$K= $\frac{E}{B_0 B}$

$$\Rightarrow {}^{q}/_{m} = {}^{K}/_{\mathrm{R}}$$

$$m\ \alpha\ R, \qquad q\ \alpha\ \frac{1}{R}$$

any:

$$q\ \uparrow \Rightarrow \quad R\ \downarrow$$

$$m\ \uparrow\ \Rightarrow \quad R\ \ \uparrow$$

Since the diameter D=2R

$$\Rightarrow R = \frac{D}{2} \quad \Rightarrow \ \frac{q}{m} = \frac{2}{DK}$$

$$\Rightarrow {}^{q}/_{m} = {}^{K'}/_{D}$$

whereas:

B: Magnetic field intensity inside the velocity filter (Tesla).

B_0: Magnetic field strength inside the velocity analyzer (Tesla).

R: radius of the ion's path (m).

Q: Charge (Coulomb).

E: Electric field strength (Volt/m).

Chapter 2:

Nuclear reactions

Chapter Two: Nuclear reactions

Atomic nucleus:

Various experiments conducted on the spherical model of the nucleus proposed by Rutherford have led to the proposal of models of the nucleus.

1- The liquid drop model: The substance (compact) is accumulated in the spherical nucleus, and resembling its structure to a liquid drop makes the nucleons play the role of molecules in the liquid, and the molecules only interact with their neighbors in the liquid body. Likewise, the bonds between the nucleons do not require the intervention of anyone except the two nucleons.

The (strong) interactions between nucleons call for nuclear forces of a very small range (1 Fermi = 10-15 metres), which overcome the Coulomb forces, which explains the stability of the positive charge concentration in the nucleus. An empirical law can be observed for all radii:

$$\mathrm{R} = \mathrm{R}_0 A^{\frac{1}{3}}$$

R0 = a constant for each Fermi element ≈ √2=√2. 10^{-15} Fermi.
R=radius of the nucleus
A=mass number

Example: Calculate the nuclide's radius ${}^{12}_{6}C$, its volumetric mass, its density, and its volumetric charge, or (its volumetric charge density)

1. Calculate the radius of the nucleus R:

$$\mathrm{R} = \mathrm{R}_0 A^{\frac{1}{3}} = \sqrt{2} \times 10^{-15} m \times (12)^{\frac{1}{3}}$$

$$R = 3{,}2 \times 10^{-15} m$$

2. Volumetric mass:

$$\rho = \frac{m}{V}$$

$$\mathrm{V} = {}^{4}/_{3}\, \pi \mathrm{R}^3 = {}^{4}/_{3}\, \pi {\mathrm{R}_0}^{3} A$$

$$V = {}^{4}/_{3}\, \pi {\mathrm{R}_0}^{3} A$$

$$m = \frac{M}{N_A} = \frac{A}{N_A}$$

$$\rho = \frac{\frac{A}{N_A}}{\frac{4}{3}\pi {R_0}^3 A} = \frac{3}{4\pi R_0^3 N_A}$$

$$\rho = \frac{3}{4 \times 3{,}1416 \times 6{,}023 \times 10^{23} \times 2{,}828427 \times 10^{-39} cm^3}$$

$$\rho = \frac{3 \qquad g}{214{,}07637 \times 10^{-16} m^3}$$

$$\rho = 1{,}4 \times 10^{14}\, {}^{g}/_{m^3}$$

$$\rho = 1{,}4 \times 10^{14} \times \frac{10^{-3} kg}{10^{-6} m^3}$$

$$\rho = 1{,}4 \times 10^{17} \frac{kg}{m^3}$$

3. Relative density:

$$d = \frac{\rho}{\rho_{H2O}}$$

$$d = \frac{1.4 \times 10^{17} \frac{kg}{m^3}}{1 \frac{g}{m^3}}$$

$$d = \frac{1.4 \times 10^{17}\, {}^{kg}/_{\mathrm{m}^2}}{1 \times \frac{10^{-3}}{10^{-6}} \quad {}^{kg}/_{m^2}}$$

$$d = 1{,}4 \times 10^{14}$$

4. Charge density:

$$\rho = \frac{+Ze}{V} = \frac{+Ze}{\frac{4}{3}\pi (R_0 A^{\frac{1}{3}})^3}$$

$$\Rightarrow \rho = \frac{3Ze}{4\pi R_0^3 A}$$

$$\rho = \frac{3 \times 6 \times 1.6 \times 10^{-19} C}{4 \times 3{,}14 \times 2{,}8284271 \times 10^{-45} \text{m}^3 \times 12}$$

$$\rho = 6{,}7 \times \frac{10^{-21}}{10^{-45}} {}^{c}/_{m^3}$$

$$\rho = 6{,}7 \times 10^{+24}\, {}^{c}/_{m^3}$$

And thus, the properties of the nucleus are as follows:

Radius: Fermi number, 1 Fermi = 10^{-15} m.

Density: 10^{14}

Volume charge density: 10^{+24} C/cm^3.

These dimensions cannot be thought about in our world, and therefore we must turn to astrophysics because in the stars there are forms of matter with this density.

2. Model of Nuclear Shell Structure:

Since the liquid drop model of the nucleus had several limitations in explaining nuclear stability and properties, physicists began to explore other models.

Since the 1950s, the shell model gradually gained prominence, but it's a highly complex model, and we won't delve into it here.

Binding Energy of Nuclei:

A. Mass-Energy Equivalence:

Physicists and chemists have discovered the principle of conservation of energy and mass.

- Antoine Lavoisier stated that in chemical reactions, there is no loss or gain of mass; instead, there is a rearrangement of particles.
- Physicists have declared that the energy of an isolated system is conserved.
- However, in the nuclear realm, the study of inelastic collisions between high-speed particles showed that:
    - Matter can convert some of its particles into kinetic energy.
    - Loss of kinetic energy can manifest as mass gain in matter.

Einstein's Relation:

The formation of a nucleus involves the components absorbing significant energy and presenting this energy as a small fraction of their mass. This is expressed by the following relation:

$$E = \Delta m C^2$$

The principle of mass-energy equivalence = **Einstein's** relationship

where:

Δm=decrease in mass

C = speed of light = 2.99979.10^8 m/s

Example: We have the following reaction:

$$N_2 + 3H_2 \rightarrow 2NH_3$$

This reaction releases an energy of 92.5 KJ for the 2 moles of ammonia formed.

1. How does this microscopic property relate to energy?
2. What is the loss in mass produced during the reaction?
3. Comment on that?

The answer

1. Thermal energy is related to microscopic kinetic energy (random movement of molecules). If the reaction releases energy, the microscopic kinetic energy of the products increases.
2. Calculate the decrease in mass Δm:

From Einstein's relationship:

$$E = \Delta m C^2$$

$$\Delta m = \frac{E}{C^2} = \frac{92.5KJ}{(2.988 \times 10^{10}\ {cm}/{s})^2}$$

$$= \frac{92.5 \times 10^3 J}{8.988 \times 10^{20}\ {cm^2}/{s^2}}$$

$$= \frac{92.5 \times 10^3 \times 10^7 erg}{8.988 \times 10^{20}\ {cm^2}/{s^2}}$$

$$1\ erg = 1g \times \frac{cm^2}{s^2} \Rightarrow \Delta m = \frac{92.5 \times 10^{10} g \times \frac{cm^2}{s^2}}{8.988 \times 10^{20}\ {cm^2}/{s^2}}$$

$$\Delta m = \frac{92.5}{8.988} \times 10^{-10} g$$

$$\Delta m = 1.03 \times 10^{-9} g$$

$$\Delta m = 1 \times 10^{-12} Kg$$

3. We note that the decrease in mass during this reaction cannot be observed with ordinary balances due to its smallness, and therefore the principle of conservation of mass proposed by Lavoisier is correct.

B. Energy equivalent of atomic mass unit:

The unit used for energy is usually the joule (J), but this unit is not adapted to fundamental particles, so we use the electron volt (eV).

Its definition: It is the energy that an electron gains when a voltage difference of 1 volt is applied to it, as:

$$1\mathrm{ev} = \mathrm{e.v} = 1{,}6 \times 10^{-19} Coulomb \times 1Volt$$

$$= V \times C^{91-}01 \times 6{,}1$$

$$\boxed{1eV = 1{,}6 \times 10^{-19} J}$$

Megaelectron volts are sometimes used:

$$\boxed{\begin{array}{c} 1MeV = 1{,}6022 \times 10^{-13} J \\ 1eV = 10^{-6}\mathrm{MeV} \end{array}}$$

From the energy equivalence equation, the value of the atomic mass unit (uma1) in electron volts can be found as follows:

$$1\mathrm{uma} = \frac{10^{-3}}{NA} KgE = \Delta m C^2$$

$$E = \frac{10^{-3}}{NA} kg \times (2{,}9978 \times 10^{10} \ \frac{m}{s})^2$$

$$= \frac{1}{NA} g \times 8{,}9978 \times 10^{20} \frac{m^2}{s^2}$$

$$= \frac{8{,}988}{6{,}023} \times \frac{10^{20}}{10^{23}} \left(\mathrm{g} \frac{\mathrm{cm}^2}{\mathrm{s}^2} \right)$$

$E = 1{,}49 \times 10^{-3} erg$ $\quad\quad$ $1J = 10^7 erg$

$E = 1{,}49 \times 10^{-3} \times 10^{-7} J$ $\quad\quad$ $1\mathrm{erg} = \frac{1}{10^7} J$

$E = 1{,}49 \times 10^{-10} J$ $\quad\quad$ $(1\mathrm{ev} = 1{,}6 \times 10^{-19} J)$

$$E = 1{,}49 \times 10^{-10} \times \frac{1eV}{1{,}6 \times 10^{-19}}$$

$$E = \frac{1{,}49}{1{,}6} \times 10^9 eV$$

$$E = 0{,}9315 \times 10^9 eV$$

$$\boxed{E(u.m.a) = 931{,}5 \times 10^6 eV}$$

$$1\mathrm{eV} = 10^{-6} MeV$$

$$\Rightarrow E(uma) = 931{,}5 \times 10^6 \times 10^{-6} Mev$$

$$931{,}5\mathrm{MeV} = \mathrm{m}C^2$$

$$931{,}5\mathrm{MeV} = 1\mathrm{u.m.a} \times C^2$$

$$\boxed{\begin{array}{c} E(uma) = 931{,}5 MeV \\ 1u.m.a = 931{,}5 \dfrac{MeV}{C^2} \end{array}}$$

Default on the nucleus mass

a. identification:

The mass of a nucleus is less than the sum of the masses of the nucleons that make it up, and we express this mathematically:

$$\boxed{\Delta m = Zm_p + (A - Z)m_n - m_{noyau} > 0}$$

When the nucleus is formed, there is a loss of mass accompanied by a release of energy that is absorbed by the components themselves, so we call:

$$E_l = \Delta m C^2$$

E_l: the binding energy of nucleons in the nucleus, which is the energy needed to break or break the cohesion of the nucleus.

$$\text{nucleus} \xrightarrow{E_l} \text{nucleons}$$

It means: expending energy to manifest the components.

b. Binding energy per nucleon:

To compare different nucleonds, the average energy of bonding of each nucleon is used and expressed mathematically:

$$f = \frac{E_l}{A} = \frac{\Delta m C^2}{A} = \left[Zm_p + (A - Z)m_n - m_{noyau}\right]\frac{C^2}{A}$$

Example: Calculate the binding energy of one nucleon of nuclide ${}^{16}_{8}O$, knowing that:

$$m_{Atome} = 15{,}9950\ u.m.a$$
$$m_p = 1{,}0073\ u.m.a$$
$$m_n = 1{,}0087\ u.m.a$$
$$m_{e-} = 5{,}5 \times 10^{-4}\ u.m.a$$

The solution:

The mass of the atom should not be confused with the mass of the nucleus if we do not neglect the mass of electrons:

$$\boldsymbol{f} = \frac{E_l}{A} = \frac{\Delta m C^2}{A} = \frac{\left[Z m_p + (A-Z)m_n - m_{noyau}\right]C^2}{A}$$

$$= \left[Z_{mp} + (A-Z)m_n - m_{atome} - z_{me}\right]\frac{C^2}{A}$$

$$= [8 \times 1{,}0073 + 8 \times 1{,}0087 - (15{,}9950 - 8 \times 5{,}5 \times 10^{-4})]\frac{C^2}{A}$$

$$= [8{,}0584 + 8{,}0696 + 2{,}2 \times 10^{-3} - 15{,}9950]\frac{C^2}{A}$$

$$f = (16{,}1302 - 15{,}9950)\frac{C^2}{16}$$

/ $1\text{uma} = \frac{931{,}5}{C^2} Mev f = \frac{0{,}1352C^2}{16} uma$

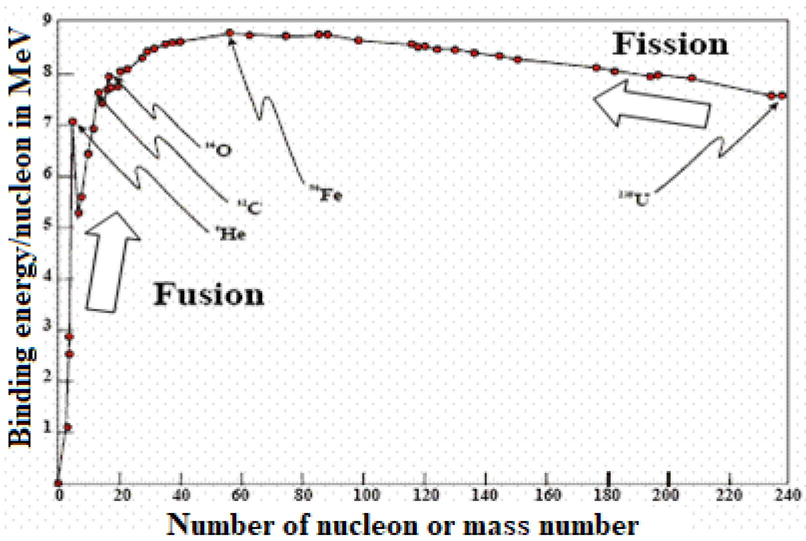
$$\Rightarrow f = \frac{0{,}1352}{16} C^2 \times \frac{931{,}5}{C^2} Mev$$

$$f = 8\ Mev$$

Radioactivity La rodioactivité

1. Aston Curve:

Aston represented the binding energy of nucleons as a function of the mass number as follows: *f=f(A)*



This curve requires several comments:

Before we start commenting on this curve, we will define radioactivity:

Definition: The nuclei of isotopes of some elements are unstable because the nucleon binding energy in them is less than 8MeV ($\frac{\Delta E}{A} < 8MeV$), and thus they collapse, giving new elements. This incoherence is accompanied by radiation, including the name radioactivity.

Nuclei in which the ratio $\frac{N}{Z} > 1,5$ are radioactive, such as uranium.

$$\frac{N}{Z} > 1,5 \quad \Longrightarrow \quad \text{Radioactive nuclei}$$

Nuclei in which the ratio $\frac{N}{Z} = 1$ or slightly more than one are stable.

$$\Longrightarrow \text{Stable nucle} \frac{N}{Z} \approx 1$$

Commentary:

a. From the graph, we can observe that when A < 30, the binding energy (f) increases, resulting in peaks. These peaks correspond to nuclei with mass numbers A = 4n, where n is an integer. These nuclei are called even-even nuclei, where Z = N = 2n.

b. When A > 30, the binding energy (f) remains relatively constant, reaching a maximum at f = 8.8 MeV, up to A = 100.

c. Then, it gradually starts decreasing, reaching f = 7.6 MeV for the nucleus ($^{238}_{92}U$).

d. This behavior leads to two types of nuclear energy production:

A. Nuclear fission:

In this process, heavy nuclei are split into lighter nuclei.

B. Nuclear fusion:

In this process, two light nuclei combine to form a more stable nucleus of the same type (even-even, for example).

$$^{1}_{1}H + ^{3}_{1}H \quad \rightarrow \quad ^{4}_{2}He + (E)$$

4 nucleons

2 protons

2 neutrons

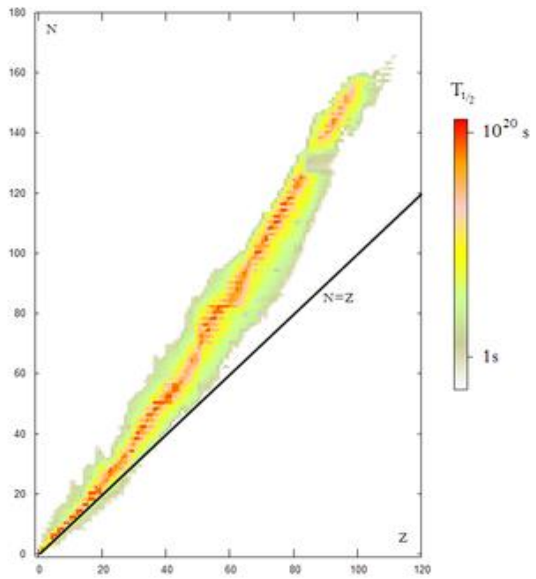
From this equation we conclude that:

$$E = f({}_{1}^{1}H) + 3f({}_{1}^{3}H) - 4f({}_{2}^{4}He)$$

$\Longrightarrow$ E<0 (It means liberated energy)

2. Stability of Nuclei:

Among the 92 natural elements, many of them contain stable isotopes (those that do not spontaneously decay), as we have seen before. These stable nuclides are referred to as natural nuclides. In addition to natural nuclides, we can obtain artificial nuclides through nuclear reactions and particle accelerators. Therefore, it is essential to plot a graphical representation of this vast number of nuclides, approximately 1500, on a curve called the stability curve, as shown in the diagram.



N=f(Z)

N = number of neutrons

Z=charge number

* **Comment on the curve:**

a. When **Z<20**, the stable nuclides are:

$$N = Z$$

$$\Rightarrow A = 2Z = 2N$$

b. When **Z > 20**, the stability zone moves away from the center towards the interior space, as:

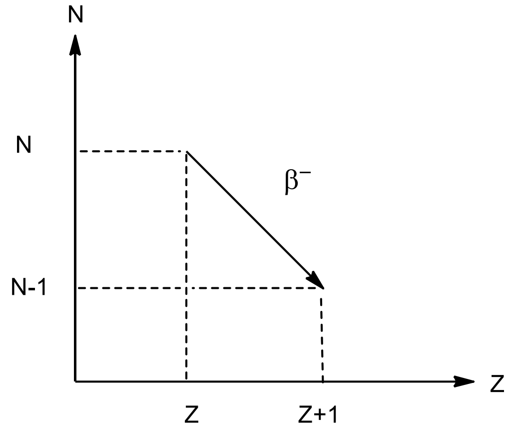
$$N > Z \Rightarrow A > 2Z$$

Therefore, unstable nuclides exist in three regions for **Z<80**:

Zone I:

The nuclide is above the stability region, and this is related to the presence of an increase in neutrons, so a neutron automatically transformation into a proton and a negative electron (-e), which is β^- rays.

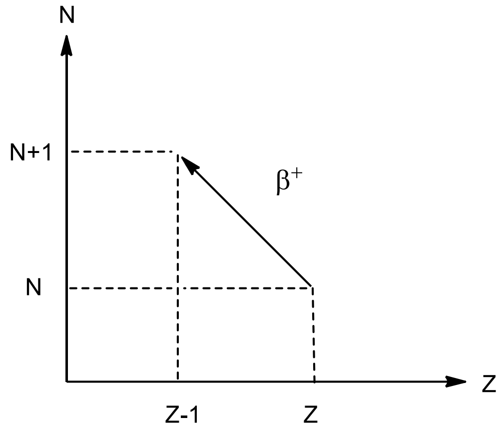
$${}_{Z}^{A}X \rightarrow {}_{Z+1}^{A}y + {}_{-1}^{0}e(\beta^-)$$



B-Zone II:

In it, the nuclide is located below the stability region, and this is related to the presence of an excess of protons, and the transformation of a proton into a neutron $({}_{0}^{1}n \leftarrow {}_{1}^{1}p)$and a positon (or positive electron) occurs, which is called β^+rays.

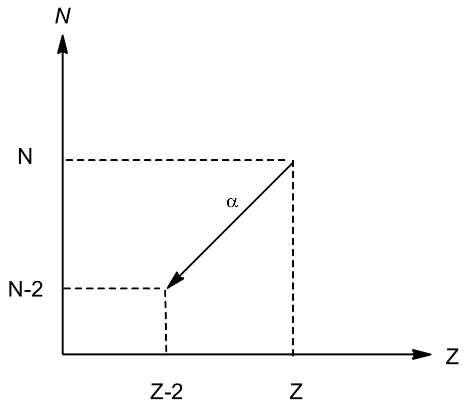
$${}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}y + {}_{+1}^{0}e(\beta^+)$$



C- Zone III:

Where they are between: 80 and up to Z=85, they are mainly natural or artificial heavy radionuclides, in which helions or rays $({}^{4}_{2}He \equiv \alpha)$ are propagated.

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He(\alpha)$$



3. Characteristics of nuclear reactions:

There are two types of nuclear reactions:

I. Spontaneous nuclear reactions

II. Induced nuclear reactions

Spontaneous nuclear reactions: in which a nucleus is automatically transformed into a nucleus or several other nuclei and a particle or several particles or radiation.

Example: $X \rightarrow Y + b$

As for induced nuclear reactions: they are reactions in which a nucleus interacts with a body called a projectile to produce another nucleus, particle, or radiation.

Example: $a + X \rightarrow Y + b$

a:
- Proton
- Neutron
- Electron
- Light nucleus
- Deutons
- Helions

b:
- α Alpha rays
- ${}^{1}_{1}p$ proton
- ${}^{1}_{0}n$ neutron
- ${}^{0}_{-1}e, {}^{0}_{1}e$ (β^{-}, β^{+})
- γ

The conservation laws:

A. There are two laws that allow us to balance nuclear reactions:

Conservation of Charge: In any nuclear reaction, the total positive and negative electric charges must remain constant.

Conservation of Nucleons: The total number of protons and neutrons in the nucleus must remain constant during the reaction.

Example of a nuclear reaction: $)^{238}_{92}U \rightarrow {}^{206}_{82}pb$ (involving a sequence of nuclear transformations through alpha (α) and beta (β^-) radiation.

To determine if this reaction is balanced in terms of charge and nucleon number, we can write the equation:

$$^{238}_{92}U \rightarrow {}^{206}_{82}pb + x\,{}^{4}_{2}He(\alpha) + y\,{}^{0}_{1}e(B^-)$$

We need to find the missing part (the unknown number) in this equation to ensure that the conservation of charge and nucleon number is satisfied.

$$238 = 206 + 4x \,\ldots\ldots(1)$$

$$92 = 82 + 2x - y \,\ldots\ldots\ldots(2)$$

From (1): $x=8$

We substitute into (2): y=6

The equation becomes:

$$^{238}_{92}U \rightarrow {}^{206}_{82}pb + 8\,{}^{4}_{2}He + 6\,({}^{0}_{-1}e)$$

B. The law of conservation of mass (mass-energy) or total energy

Which allows the definition of nuclear energy

$$\boxed{E_{total} = E_c + \Delta m C^2}$$

in which:

$$a + x \rightarrow y + b$$

$$Ec_{(a)} + \Delta m_{(a)}C^2 + Ec_{(x)} + \Delta m_{(x)}C^2 = Ec_{(y)} + \Delta m_{(y)}C^2 + Ec_{(b)} + \Delta m_{(b)}C^2$$

By definition:

$$Q = \left(m_y + m_b - m_a - m_x\right)C^2 = \Delta m C^2$$

It results that :

$$Q = Ec_{(a)} + Ec_{(x)} - Ec_{(y)} - Ec_{(b)}$$

$$\boxed{Q = -\Delta Ec}$$

The physical meaning of this :

a. When Δm < 0, there is a loss of matter, meaning Q < 0, and as a result, energy is released, which is taken by the surrounding material in the form of kinetic energy (ΔEc > 0). This leads to spontaneous reactions, meaning reactions that occur naturally.
b. Conversely, when Δm > 0, there is a gain in matter, implying Q > 0. In this case, there is an absorption of energy provided by the incoming particles with high kinetic energy. Therefore, the reaction is non-spontaneous, meaning it doesn't occur naturally.

I. Spontaneous nuclear reactions:

A. Historical Overview:

The phenomenon of natural radioactive activity was discovered by the scientist Becquerel when he observed that photographic plates were affected when placed near uranium salts (K2UO2(SO4).2H2O).

By using an electric field, researchers were able to categorize nuclear radiations into three main types:

- Undeflected rays under the influence of the electric field were called gamma rays (γ), which are electromagnetic radiations.
- Two other beams are deflected in opposite directions: positive α rays and negative β^- beta rays.

- The properties of these alpha, beta, and gamma rays were studied by Pierre and Marie Curie, and further advancements in these studies were made by Irene and Frederic Joliot-Curie in 1934.

$${}^{27}_{13}Al + {}^{4}_{2}He \rightarrow {}^{30}_{15}P^* + {}^{1}_{0}n \qquad \text{(reaction provoked)}$$

Whereas ${}^{30}_{15}P^*$ an artificial radionuclide automatically gives off a new type of radiation, which is β+ radiation:

$$^{30}_{15}P^{*} \rightarrow ^{30}_{14}Si + ^{0}_{1}e \ \ (\beta^{+})$$

Radiation ($^{4}_{2}He$≡α) :

α rays are helions $^{4}_{2}He^{++}$helium nuclei, which are observed with heavy nuclei and are subject to the following equation :

$$^{A}_{Z}X \rightarrow ^{A-4}_{Z-2}y + ^{4}_{2}He \text{ أو } (^{4}_{2}\alpha)$$

Note: Nuclear reactions occur with atoms, but at the level of nuclei.

Some quantities and dimensions:

- The nuclear energies of alpha interactions are of the order of a few MeV.
- The fastest particle or ray α has a speed of about $1{,}5.10^{7}\ ^{m}/_{s}$.
- Despite this high speed, α-rays are easily stopped by matter.

	Live weave	Lead	air
α (1MeV)	8μm	1μm	0,5m

C- Radiation (β) (β+, β-):

Definition: It is radiation related to the emission of an electron or its antiparticle, a positron. In the case of natural radionuclides, we practically observe the emission of β^{-} and antineutrino radiation. It is also infused into synthetic heavy elements, as in the equation:

$$^{A}_{Z}X \rightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}e + (^{0}_{0}\nu^{-}) \rightarrow antineutrino$$

In this reaction, a neutron in the nucleus turns into a proton and also emits β^{-} radiation and a particle called an antiparticle or antineutrino, as in the equation:

$$^{A}_{Z}X \rightarrow {}_{Z-1}^{A}Y + {}_{+1}^{0}e^{+} + (^{0}_{0}\nu^{+}) \rightarrow neutrino$$

A proton is converted into a neutron according to this reaction:

$$^{1}_{1}p \rightarrow ^{1}_{0}n + {}_{+1}^{0}e + (^{0}_{0}\nu^{+})$$

Note: The neutrino was discovered by Pauli in 1931 when he calculated the energy associated with β decay. He proposed the existence of a very light, uncharged particle that is also emitted along with β rays, and it was named the neutrino. Its energy is at its maximum when the β energy is at its minimum $w_0 = \beta + ^{0}_{0}\nu$.

Some characteristics of β rays:

- The nuclear energies of β (β^+, β^-) reactions are on the order of a few MeV.
- Electrons or positrons have very small masses compared to helium nuclei (^4_2He), and their velocities can reach up to 0.99C, where C is the speed of light.
- β rays are more penetrating than α rays.

	Live weave	Lead	air
β (1MeV)	4mm	0 ,33mm	2,9m

d- Gamma Radiation (γ):

Gamma radiation consists of electromagnetic rays of the same nature as visible light. They have very short wavelengths and are emitted when a nucleus, Y*, transitions from an excited state caused by α or β emissions back to its ground state:

$$Y^* \rightarrow Y + \gamma$$
$$W = hv = E_Y^* - E_Y$$

The resulting new nucleus is in an excited state, and it must rearrange its nucleons to reach a more stable state. The energy released in the movements of protons and neutrons within the nucleus is emitted in the form of radiation, similar to the transition of electrons in an atom.

Some dimensions of gamma rays:

Gamma rays have a wavelength of $\lambda = 10^{-12}m$ meters and high penetration ability compared to β and α rays.

	Live weave	Lead	air
(1MeV)γ	15cm	1,5cm	150m

Its uses(γ):

1. **In radiation therapy:** it is used to kill cancer cells.
2. **In Gammagraphie,** it is used to measure density and study the structure of solid bodies.

The motion of spontaneous nuclear reactions:

Non-radioactive resulting nucleus:

Law of Decay:

Let's assume that X is a radioactive element "parent" that undergoes decay to produce element Y "daughter," as shown in the equation:

	X	$\rightarrow$ Y	
$t = 0$	N_0	0	At moment (0) there is N0 radioactive nucleus
$t = t$	N	$N_0 - N$	

Where: N is the number of unstable nuclei or atoms of X remaining at moment t.

N0-N is the number of stable Y atoms formed.

At the moment t+dt, N decreases by dN, where dN is the number of unstable nuclei that collapsed during the period dt or the number of stable nuclei formed during the period dt.

According to Soddy's law, which states that "the probability of X turning into Y in a small moment dt is equal to λdt."

Where λ : is the radioactivity constant and represents the probability of collapse per second.

We define the speed of collapse as the number of collapse per unit time:

dN : the number of collapsing nuclei in an instant dt $-\frac{dN}{dt} = +\lambda N$

The sign (-) indicates a decrease in the substance.

$$\Rightarrow \frac{dN}{N} = -\lambda dt \quad \Rightarrow \quad \int \frac{dN}{N} = -\lambda \int dt$$

$$\ln N = -\lambda t + \ln c$$

When: t=0, it is: N=N0

$$\ln N_0 = \lambda \times 0 + \ln c$$

$$\Rightarrow \ln N_0 = \ln c \Rightarrow N_0 = c$$

We substitute the value of c into the previous equation:

$$\ln N = -\lambda t + \ln N_0$$

$$\Rightarrow \frac{N}{N_0} = e^{-\lambda t} \qquad \Rightarrow \ln \frac{N}{N_0} = -\lambda t$$

$$\boxed{N = N_0 e^{-\lambda t}}$$

This means that the nu ... ver time.

λ is the radioactive decay constant, and it changes with a change in A (activity).

The half-life, often represented by the symbol "T," which is the time it takes for half of the nuclei X to transform into Y, and its unit is seconds (s).

when t=T : $N = \frac{N_0}{2}$

We substitute the previous equation:

$$ln\frac{N}{N_0} = -\lambda t \Rightarrow ln = \frac{N_0}{2\ N_0} - \lambda t$$

$$\Rightarrow -ln2 = -\lambda T \Rightarrow ln\frac{1}{2} = -\lambda t$$

$$T = \frac{ln2}{\lambda} = \frac{2{,}3 \times 0{,}3}{\lambda}$$

$$\boxed{T = \frac{0{,}69}{\lambda}}$$

According to the radionuclide, the half-life extends from approximately 10^{10} years to 10^{-7} seconds.

For example:

$$^{212}_{84}Po : T = 3 \times 10^{-7}s$$

$$^{222}_{86}Rn : T = 3.8\ j$$

$$^{232}_{90}Th : T = 1.4 \times 10^{10} years$$

Graphical representation of the relationship: $N = N_0 e^{-\lambda t}$

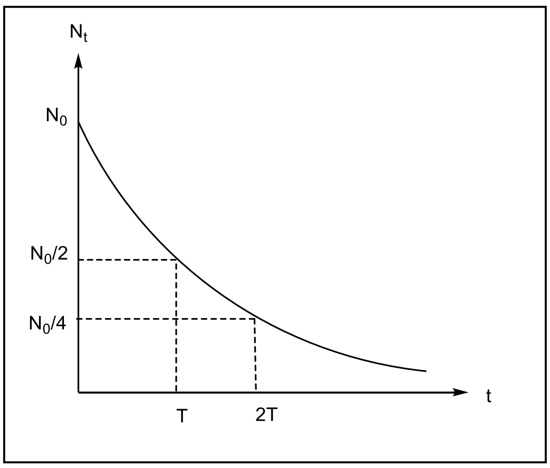
$$t = 0 \Rightarrow N = N_0$$

$$t = T \Rightarrow N = \frac{N_0}{2}$$

$$t = 2T \Rightarrow N = \frac{N_0}{4}$$

$$t = 3T \Rightarrow N = \frac{N_0}{8}$$

$$t = nT \Rightarrow N = \frac{N_0}{2^n}$$



Activity:

Activity is the number of decay per unit time given by definition by the following relationship:

$$\boxed{A = -\frac{dN}{dT} = +\lambda N}$$

as:

$$N = N_0 e^{-\lambda t}$$

$$A = \lambda . N_0 e^{-\lambda t}$$

If we put: $A_0 = \lambda N_0$

$$\boxed{\Rightarrow A = A_0 e^{-\lambda t}}$$

The unit of activity in the international system is the becquerel:

$$\mathrm{Becquerel} = \mathrm{Bq} = \frac{\mathrm{decqy}}{\mathrm{sec}}$$

This unit has replaced another unit, Curie, with its symbol (Ci), where :

$$1\mathrm{Ci}=3.7.10^{10}\ \mathrm{Bq}$$

Example: What are the masses of $^{226}_{88}Ra$ (T = 1590ans) and $^{137}_{56}Ba$ $(T = 2{,}6min)$, knowing that their activity is equal to 1Bq

The solution

Assuming that the mass number A = atomic mass M.

Number of transformed nuclei in time t:

Since the activity:

$$A = \lambda N$$

$$\Rightarrow A = \lambda \frac{m}{M} N_A$$

Since the: $T = \frac{ln2}{\lambda}$ $\Rightarrow$ $\lambda = \frac{ln2}{T}$

We substitute into the previous relationship:

$$\Rightarrow A = \frac{ln2}{T}\frac{m}{M} N_A$$

$$\Rightarrow m = \frac{A.M.T}{N_A.\ln 2}$$

If the activity = $\frac{des}{s} = 1Bq$

The half-life time T must be = in seconds (s). By numerical substitution it is:

$$m_{Ra} = \frac{1^{des}/_{s} \times 226g \times 5{,}01422\times10^{10}s}{6{,}023\times10^{23}\times0{,}693}$$

$$m_{Ra} = 2.7 \times 10^{-11}g$$

$$m_{Ba} = \frac{1^{des}/_{s} \times 137g \times 2{,}6\times60s}{6{,}023\times10^{23}\times0{,}693}$$

$$g^{02-}01 \times 21{,}5 = {}_{aB}m$$

Note: During the decay of a nucleus, the resulting nucleus must be radioactive or can give rise to an unstable nucleus, and thus, we have a series of nuclei that appear one after another until it reaches a stable nucleus. This collection of nuclei from the parent to the last grandchild forms a family.

The radioactive families:

There are four radioactive families, three of them are natural, and the fourth one is artificial, and these families are:

1. **Thorium Family $^{232}_{90}Th$**:

   In this family, 6 alpha particles and 4 beta-minus particles are emitted, as shown in the following equation:

$$^{232}_{90}Th \rightarrow\ ^{208}_{82}Pb + x\,^{4}_{2}He + y(^{\ 0}_{-1}e \equiv \beta^-)$$

$$\begin{cases} 232 = 208 + 4x + 0 \\ 90 = 82 + 2x - y \end{cases}$$

$$\begin{cases} x = \dfrac{232 - 208}{4} = \dfrac{24}{4} = 6 \Rightarrow x = 6 \\ -y = 90 - 82 - 12 = -4 \Rightarrow y = 4 \end{cases}$$

So the equation becomes the following:

$$^{232}_{90}Th \rightarrow \ ^{208}_{82}Pb + 6^4_2He + 4(^{\ 0}_{-1}e \equiv \beta^-)$$

$$\Rightarrow A = 4n$$

$$\Rightarrow n = 58 \quad \rightarrow \quad 52$$

Where n is a natural number:

Number of father's mass: $A = 4 \times 58 = 232$

Number of Son Block: $A = 4 \times 52 = 208$

2. Plutonium Family $^{241}_{94}Pu$:

This is the only artificial radioactive family obtained from plutonium after its discovery and separation from uranium -238. The decay of plutonium produces 8 alpha particles and 5 beta-minus particles, as shown in the following equation:

$$^{241}_{94}Pu \rightarrow \ ^{209}_{83}Bi + x^4_2He + y(^{\ 0}_{-1}e \equiv \beta^-)$$

$$\begin{cases} 241 = 209 + 4x + 0 \\ \ 94 = 83 + 2x - y \end{cases}$$

$$\begin{cases} x = \dfrac{241 - 209}{4} = \dfrac{32}{4} = 8 \ \Rightarrow \ x = 8 \\ -y = 94 - 83 - 16 = -5 \ \Rightarrow \ y = 5 \end{cases}$$

So the equation becomes the following:

$$^{241}_{94}Pu \rightarrow \ ^{209}_{83}Bi + 8^4_2He + 5(^{\ 0}_{-1}e \equiv \beta^-)$$

$$\Rightarrow A = 4n + 1$$

$$\Rightarrow n = 60 \quad \rightarrow \quad 52$$

Where n is a natural number:

Number of father's mass: $A = 4 \times 60 + 1 = 241$

Number of Son Block: $A = 4 \times 52 + 1 = 209$

3. Uranium Family ($^{238}_{92}U$):

This is a radioactive family in which 8 alpha particles and 6 beta-minus particles are emitted, as shown in the following equation:

$$^{238}_{92}U \rightarrow \; ^{206}_{82}Pb + x\,^{4}_{2}He + y(^{0}_{-1}e \equiv \beta^{-})$$

$$\begin{cases} 238 = 206 + 4x + 0 \\ \;\; 92 = 82 + 2x - y \end{cases}$$

$$\begin{cases} x = \dfrac{238 - 206}{4} = \dfrac{32}{4} = 8 \;\Rightarrow\; x = 8 \\ -y = 92 - 82 - 16 = -6 \;\;\Rightarrow\;\; y = 6 \end{cases}$$

So the equation becomes the following:

$$^{238}_{92}U \rightarrow \; ^{206}_{82}Pb + 8\,^{4}_{2}He + 6(^{0}_{-1}e \equiv \beta^{-})$$

$$\Rightarrow A = 4n + 2$$

$$\Rightarrow n = 60 \quad \rightarrow \quad 52$$

Where n is a natural number:

Number of father's mass: $A = 4 \times 59 + 2 = 238$

Number of Son Block: $A = 4 \times 51 + 1 = 206$

4. Uranium Family $^{235}_{92}\boldsymbol{U}$:

The final family (also known as the Actinium family) is the Uranium $^{235}_{92}\boldsymbol{U}$ family, comprising 0.71% of naturally occurring uranium. In its decay, it emits 7 alpha particles and 4 beta-minus particles, as shown in the following equation:

$$^{235}_{92}U \rightarrow \; ^{207}_{82}Pb + x\,^{4}_{2}He + y(^{0}_{-1}e \equiv \beta^{-})$$

$$\begin{cases} 235 = 207 + 4x + 0 \\ \;\; 92 = 82 + 2x - y \end{cases}$$

$$\begin{cases} x = \dfrac{235 - 207}{4} = \dfrac{28}{4} = 7 \Rightarrow x = 7 \\ -y = 92 - 82 - 14 = -4 \Rightarrow y = 4 \end{cases}$$

So the equation becomes the following:

$$^{235}_{92}U \rightarrow {}^{207}_{82}Pb + 7\,^{4}_{2}He + 4(^{0}_{-1}e \equiv \beta^-)$$

$$\Rightarrow A = 4n + 3$$

$$\Rightarrow n = 58 \quad \rightarrow \quad 51$$

Where n is a natural number:

Number of father's mass: $A = 4 \times 58 + 3 = 235$

Number of Son Block: $A = 4 \times 51 + 3 = 207$

If we compare the roles of the elements (parent) or generators in each family, we will understand why the Plutonium Family $^{241}_{94}Pu$ is an artificial family.

$$T_{(Th-232)} = 1{,}4 \times 10^{10}\ ans$$
$$T_{(U-238)} = 4{,}5 \times 10^{9}\ ans$$
$$T_{(U-235)} = 7{,}1 \times 10^{8}\ ans$$
$$T_{(Pu-241)} = 14{,}7\ ans$$

In a radioactive nuclear family resulting from a parent nucleus, the offspring have very small roles and are thus in a gamma-ray equilibrium.

Provoked Nuclear Reactions:

A. Transmutation Reactions:

Definition: These are reactions that produce nuclei with a mass number approximately equal to or very close to the mass number of the nucleus used as a target.

The resulting nuclei can be either stable or radioactive, and the first reaction of this type was observed by Rutherford in 1919.

$$^{4}_{2}He + {}^{14}_{7}N \rightarrow {}^{17}_{8}O + {}^{1}_{1}H(^{1}_{1}p)$$

This is a (α, p) reaction and can be written symbolically as:

$$^{14}_{7}N(\alpha,p)^{17}_{8}O$$

Julius Kell observed that the resulting nuclei are themselves radioactive. This marks the beginning of the discovery of β+ radiation.

Reaction ((α, n):

$$^{4}_{2}He + ^{27}_{13}Al \rightarrow ^{30}_{15}P^{*} + ^{1}_{0}n$$
$$\hookrightarrow ^{30}_{14}Si + (^{0}_{+1}e \equiv \beta^{+}) + ^{0}_{0}\nu$$

$$\Rightarrow ^{27}_{13}Al(\alpha,n)^{30}_{15}P^{*}$$

There are other examples.

We can divide these reactions into:

1. Reactions with Helium Nuclei:

- **Reactions ((α, p):**

$$^{4}_{2}He + ^{10}_{5}B \rightarrow ^{13}_{6}C^{*} + (^{1}_{1}H \equiv ^{1}_{1}p) \quad -$$
$$\Rightarrow ^{10}_{5}B(\alpha,p)\ ^{13}_{6}C^{*} \quad -$$

And it is written as follows:

$$^{4}_{2}\alpha + ^{A}_{Z}X \rightarrow ^{A+3}_{Z+1}Y^{*} + ^{1}_{1}p$$

- **Reactions (α, n):**

$$^{4}_{2}He + ^{27}_{13}Al \rightarrow ^{30}_{15}P^{*} + ^{1}_{0}n \Rightarrow$$
$$- \quad ^{27}_{13}Al(\alpha,n)^{30}_{15}P^{*}$$
$$^{4}_{2}He + ^{10}_{5}B \rightarrow ^{13}_{7}N^{*} + ^{1}_{0}n \Rightarrow$$
$$^{10}_{5}B(\alpha,n)^{13}_{7}N^{*} \quad -$$

And the symbolic notation for these reactions is as follows:

$$^{4}_{2}\alpha + ^{A}_{Z}X \rightarrow ^{A+3}_{Z+1}Y^{*} + ^{1}_{0}n$$

The resulting nuclei are themselves radioactive, and as long as they are radioactive, they emit β+ particles.

- **Reactions Without Capture:**

In these reactions, alpha rays are used as carriers of energy only. For example:

$$ {}_{2}^{4}He + {}_{5}^{10}B \rightarrow {}_{1}^{1}H + {}_{4}^{9}Be + {}_{2}^{4}He $$

2. Reactions with Protons:

- Among these are reactions of the type **(p,α),** in which protons have high speed.

$$ {}_{1}^{1}H + {}_{7}^{14}N \rightarrow {}_{6}^{11}C^{*} + {}_{2}^{4}He $$

$$ {}_{7}^{14}N(\mathrm{p},\alpha){}_{6}^{11}C^{*} $$

These reactions release a lot of heat.

- **(p,n) Reactions:**

These are reactions in which protons have very high speed

$$ {}_{1}^{1}H + {}_{29}^{63}Cu \rightarrow {}_{30}^{63}Zn + {}_{0}^{1}n $$

$$ {}_{29}^{63}Cu(\mathrm{p},n){}_{30}^{63}Zn $$

- Reactions where protons are captured or captured

$$ {}_{1}^{2}H + {}_{4}^{9}Be \rightarrow {}_{5}^{10}B \Rightarrow {}_{4}^{9}Be(p){}_{5}^{10}B $$

$$ {}_{1}^{1}H + {}_{8}^{16}O \rightarrow {}_{9}^{17}F^{*} + {}_{8}^{17}O + ({}_{+1}^{0}e \equiv \beta^{+}) $$

$$ {}_{8}^{16}O(\mathrm{p}){}_{9}^{17}F^{*} $$

- **(p,d)** Reactions, which are very rare:

$$ {}_{1}^{1}H + {}_{4}^{9}Be \rightarrow {}_{4}^{8}Be + {}_{1}^{2}H(Deutron) $$

$$ {}_{4}^{9}Be(\mathrm{p},\mathrm{d}){}_{4}^{8}Be $$

3. Reactions with Deuterons:

$$ \nearrow {}_{6}^{11}C + {}_{0}^{1}n $$

$$ {}_{1}^{2}H + {}_{5}^{10}B \rightarrow {}_{5}^{11}B + {}_{1}^{1}H $$

$$ \searrow 3\,({}_{2}^{4}He) $$

$$ {}_{1}^{2}H + {}_{83}^{209}Bi \rightarrow {}_{83}^{210}Bi + {}_{1}^{1}H $$

4. Reactions with Neutrons:

Neutron Capture:

$$ {}^{1}_{0}n + {}^{79}_{85}Br \rightarrow {}^{80}_{35}B^{*} $$

Neutron capture becomes easier as the neutron speed decreases.

$$ {}^{1}_{0}n + {}^{238}_{92}U \rightarrow {}^{239}_{92}U^{*} \rightarrow {}^{239}_{93}Np^{*} + ({}^{0}_{-1}e \equiv \beta^{-}) $$

$$ \downarrow $$

$$ {}^{239}_{94}Pu + ({}^{0}_{-1}e \equiv \beta^{-}) $$

Note that neutron capture allows us to produce heavy radioactive elements with Z>92.

- **(n, p) Reactions:**

$$ {}^{1}_{0}n + {}^{31}_{15}P \rightarrow {}^{1}_{1}H + {}^{31}_{14}Si^{*} \Rightarrow {}^{31}_{15}P(n,p){}^{31}_{14}Si^{*} $$

- **(n, n) Reactions:**

  The bombardment of heavy nuclei leads to the fission of these nuclei into two or several nuclei with an average charge number, and these reactions are accompanied by immense energy.

This means nuclear fission with the production of other neutrons as follows:

$$ {}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{143}_{56}Ba + {}^{83}_{36}Kr + 10\,{}^{1}_{0}n $$

And we note that transmutation reactions have allowed us to extend the periodic table of elements by synthesizing new elements with Z>92, known as transuranium elements or post-uranium elements. All of these elements are synthetic, and some of them have important properties, such as (${}^{239}_{94}Pu$)

Nuclear Fusion:

These are reactions in which two light nuclei combine to form a heavier nucleus, generally isotopes of hydrogen, in a single nucleus. This reaction releases immense energy, as in the following example:

$$ {}^{1}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n \quad (\Delta m < 0, Q < 0) $$

This is the reaction used in a hydrogen bomb (it's a fusion that cannot be controlled). To achieve this nuclear fusion, the repulsive force between the nuclei must be overcome to make them collide. This requires enormous energy, extreme heat (106°C), which can only be provided by a nuclear bomb (Bombe A), and it's the only one that can generate this heat. This is why it's used as the trigger for a hydrogen bomb (Bomba H) to create the explosion.

Properties of Transmutation Reactions:

These reactions allow the production of natural radioactive nuclides for each element. In particular, reactions of the radiative capture type (n,γ) in the nuclear families are very important, and their equation is:

$$ {}_{Z}^{A}X + {}_{0}^{1}n \rightarrow {}^{A+1}_{\ \ \ Z}X^{*} + \gamma $$

When neutrons are fast, deeper transformations of the (n, p) or (n, α) type occur.

The importance of tracers continues to grow day by day:

In Chemistry:

- They are used in labeling techniques to study the mechanisms of organic chemical reactions.
- They are used in isotopic dilution calibration and provide valuable information (Dosage par dilution isotopique).

In Biology:

- They are used to study the metabolism of most elements (Na, K, Ca, P).
- They are used to measure blood volume using the isotopic dilution method.
- Calibration of growth hormones like insulin in the blood.

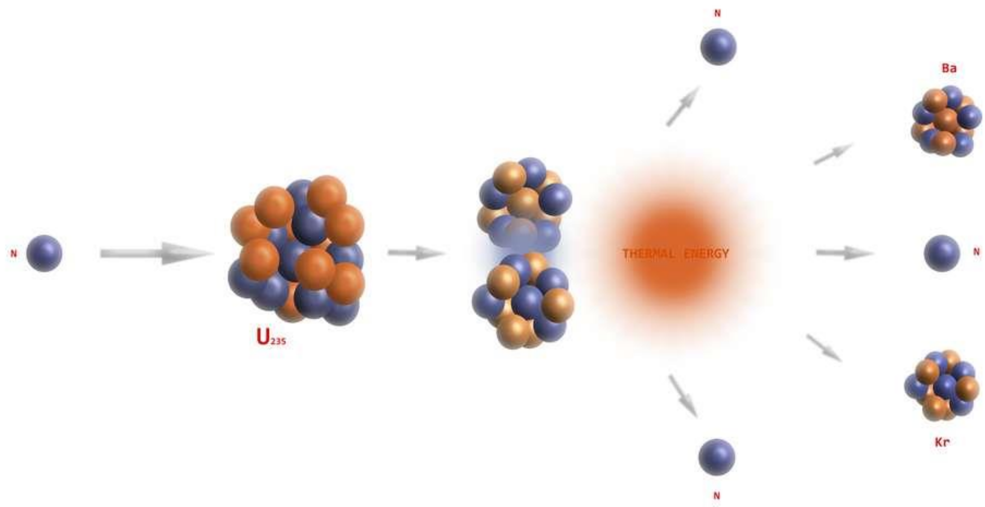
In Medicine:

- They are used to locate tumors with the help of radioactive tracers (Traceurs radioactif).
- They are used in scintigraphy: capturing images of an organ by emitting radiation through a radioactive nucleus embedded in it.
- Treating diseased cells metabolically by attaching a radioactive nucleide for (β^-) radiation. In this case NaI^* $({}^{131}I^*)$ is used, which enters the blood and settles in the thyroid gland, contributing to the formation of thyroid hormones.

Nuclear Fission:

Under neutron bombardment, a heavy nucleus undergoes fission, splitting into two lighter nuclei. However, these two nuclei are obtained in their excited states, and they, in turn, emit or release neutrons, leading to the possibility of a nuclear chain reaction.

As shown in the diagram:



The result of nuclear fission in the case of uranium $^{235}_{92}U$, for example, can potentially yield all elements between zinc Zn (A=65) and dysprosium $^{163}_{66}Dy$ (A=163).

Additionally, there can be emission of 1, 2, 3, or 4 neutrons.

The average released energy per atom of uranium$(^{235}_{92}U)$ is a staggering 200 MeV.

Example: Consider the following reaction:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow \ ^{146}_{57}La + ^{87}_{35}Br + 3^{1}_{0}n$$

Where the masses of the nuclei are as follows:

$$^{235}_{92}U = 235{,}044\mathrm{u.m.a}$$
$$^{146}_{57}La = 145{,}943\mathrm{u.m.a}$$
$$^{87}_{35}Br = 86{,}912\mathrm{u.m.a}$$

Neutron Mass:

$$^{1}_{0}n = 1{,}0087\mathrm{u.m.a}$$

Requested:

1. Calculate the energy released per atom of uranium$(^{235}_{92}U)$ in MeV.
2. Calculate the energy released by 1 mole of uranium$(^{235}_{92}U)$ atoms in joules.

Solution:

1. By definition:

$$Q = \Delta m C^2$$

$$Q = m_{La} + m_{Br} + 3m_n - m_n - m_U)C^2$$

$$Q = m_{La} + m_{Br} + 2m_n - m_U)C^2$$

But :

$$1(\mathrm{u\,.m.a}) = \frac{931{,}5MeV}{c^2}$$

$$Q = 931{,}5(145{,}943 + 86{,}912 + 2 \times 1{,}0087 - 235{,}044)\frac{c^2}{c^2}$$

$$= 931{,}5(234{,}8724 - 235{,}044)Me$$

$$= 931{,}5 \times (-0{,}172)MeV$$

$$\boxed{Q = -160MeV}$$

It is the energy released by one atom of uranium.

2. The energy released by 1 mole of uranium atoms:

$$\mathrm{Q = -160MeV \times N_A}$$

That is, we multiply by Avogadro's number for one mole.

$$Q = -160 \times 6{,}023 \times 10^{23}$$

$$Q = -9{,}63 \times 10^{25} Mev$$

$$1MeV = 1{,}6 \times 10^{-13} J$$

$$Q = -9{,}63 \times 10^{25} \times 1{,}6 \times 10^{-13}\ J$$

$$\boxed{Q = -1{,}54 \times 10^{13} {}^{J}/_{mol}}$$

It is a significant value compared to the dimensions of energy released during chemical reactions such as combustion reactions (10^6 J/mol).

Chapter 3:

Difficulties of the Rutherford Model

Chapter Three
Difficulties of the Rutherford Model

In the Rutherford atomic model, we observed a complete contradiction to the previous Thomson model. As mentioned earlier, Rutherford attempted to validate the Thomson model and made significant strides in atomic theory. However, this model required further refinement for several reasons:

- We know nothing about the behavior of electrons around the nucleus.
- How they exist at a distance from the nucleus while they should be attracted to it.
- Since electrons don't fall into the nucleus, it is believed that they are in circular motion around the nucleus, ensuring a stable position at a specific distance. This was seemingly easy to demonstrate.
- However, according to the electromagnetic theory known at that time, which stated that "every motion of an electric charge is accompanied by the emission of electromagnetic radiation, and this radiation results in a loss of the charge's energy, ceasing only when the charge stops moving."
- This made the electron's position difficult to determine; it could either be:

(1) Stationary (i.e., motionless), but it would fall into the nucleus, contradicting Rutherford's experimental findings.

(2) Moving while simultaneously emitting radiation, leading to energy loss and eventual collapse into the nucleus.

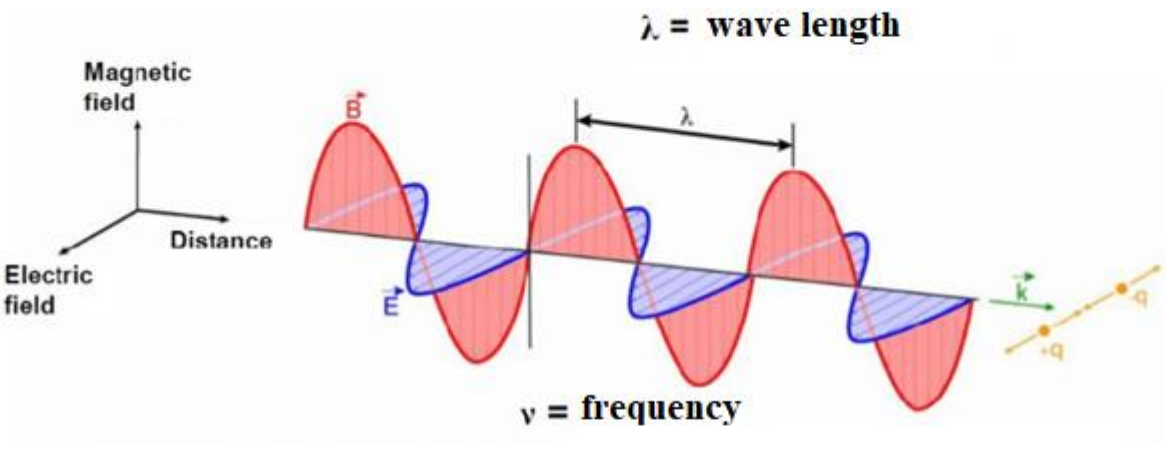
To solve this dilemma, a new theory had to be adopted, and this theory is called "Quantum Theory of Energy" (Théorie de la quantification de l'énergie), which was first proposed by Planck in 1900 and later developed by Einstein in 1905.

Energy quantification:

The dual nature of light:

A. Wave nature:

Light waves belong to the electromagnetic waves, which are associated with the propagation of two perpendicular fields, one being electric ($\vec{E}$) and the other magnetic ($\vec{B}$), both of which vary sinusoidally with time ($\vec{B}$, $\vec{E}$).



The continuous wave propagates in a vacuum at the speed of light ($c=3\times10^{8}$ m/s) and can be characterized by either:

Its frequency $\boldsymbol{\nu = \frac{1}{T}}$, where ν represents frequency.

Or its wavelength λ, where $\boldsymbol{\lambda = cT = \frac{c}{\nu}}$

In the electromagnetic spectrum, visible light for the human eye corresponds to a narrow range of wavelengths $\lambda \in [400nm, 770nm]$.

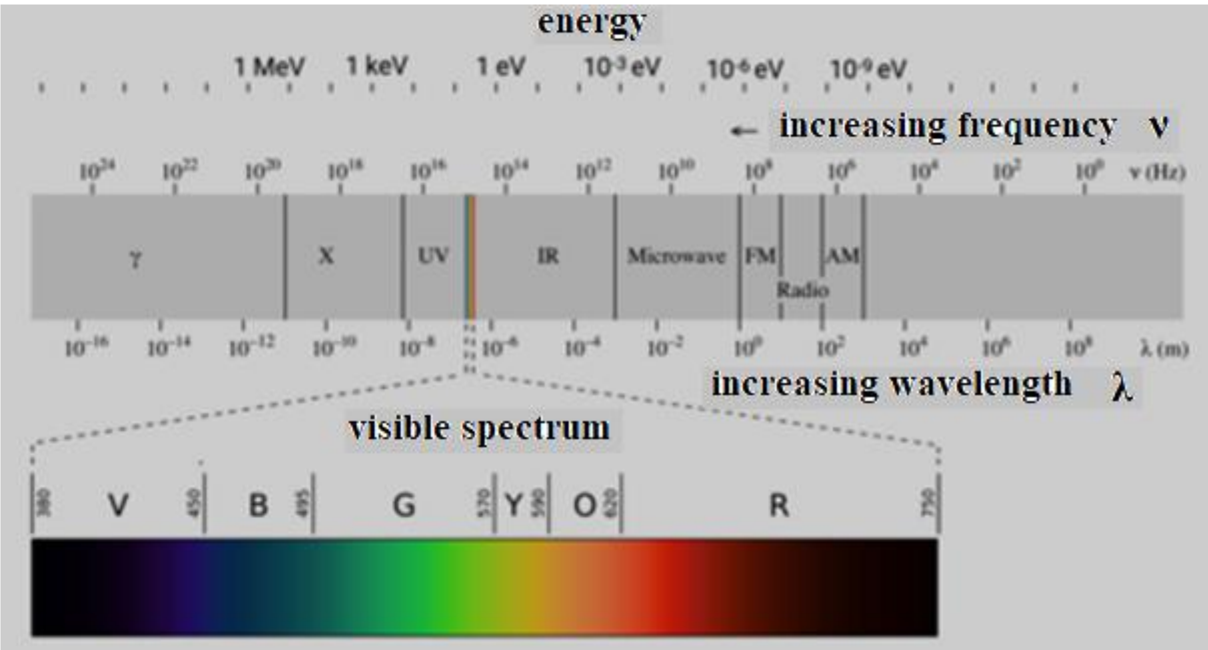
Where:

c = the speed of light in a vacuum.

ν = frequency (in Hertz).

T = period (in seconds).

λ = wavelength (in meters or centimeters).



B. Corpuscular Nature

The wave nature of light, characterized by phenomena like interference and refraction, does not allow for an explanation of discontinuous energy exchange. Therefore, it cannot explain phenomena such as the photoelectric effect.

C. Duality (Wave-Particle Duality)

The scattered photon carries an amount of energy given by:

$$E = h\nu = h\frac{c}{\lambda} \quad \text{-------} \quad 1$$

According to Einstein's mass-energy equivalence, energy can be expressed as:

$$\text{-------} \quad 2 \qquad\qquad E = mc^2$$

Setting equation 2 equal to 1:

$$\Rightarrow h\frac{c}{\lambda} = mc^2$$

Where: P = momentum.

$$v = c \qquad\qquad P = mv = mc$$

From this, the photon that propagates at the speed of light has a wavelength given by:

$$\lambda = \frac{h}{mc} = \frac{h}{p}$$

Note: m is the mass that appears when energy E is converted into matter, and conversely, when a quantity of matter with mass (m) disappears, it results in energy E that is equivalent to the mass (m).

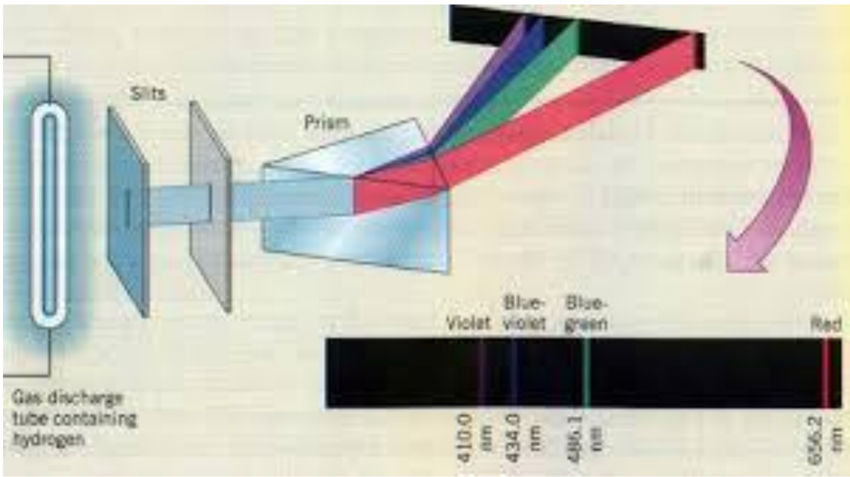
Atomic Model of Bohr

We have seen some difficulties in Rutherford's atomic model, where the electron occupies positions around the nucleus. It can either fall into the nucleus if it's motionless or emit electromagnetic radiation if it's in motion, ultimately falling into the nucleus.

To address this dilemma, a new theory was needed, as we have seen, and that theory is Planck's Quantum Theory. Before delving into Bohr's theory, let's briefly recap the experimental results explained by Bohr's assumptions, which contributed to the success of Bohr's theory.

Hydrogen Spectra and the Balmer Formula

An apparatus called a Geissler tube is used. It contains a capillary tube in the center and two broad ends, as shown in the figure.



- This tube contains hydrogen gas at low pressure.
- At both ends of the tube, electrodes are attached. When the electrodes are connected to a high voltage generator, the hydrogen inside the tube becomes luminous due to the ionization of its molecules.
- To analyze the light emitted by hydrogen, we pass this light through a glass prism.
- This glass prism has the property of breaking down natural light into single-colored radiations. Since the prism separates white light into a range of colors forming a rainbow.

- This means that colors with different wavelengths do not undergo the same deviation when passing through the prism. Consequently, the radiation emitted by hydrogen gas, when placed in front of a glass prism, gets separated into a range of colors.

The collection of these colors obtained on the photographic plate is called the hydrogen gas spectrum. This spectrum is continuous because the colors gradually change without interruption.

1- A red line with a wavelength of λ = 6563 A°
2- A blue line with a wavelength of λ = 4861 A°
3- A cyan line with a wavelength of λ = 4340 A°
4- A violet line with a wavelength of λ = 4102 A°

The combination of these spectral lines on the photographic plate forms the visible line spectrum of hydrogen.

This spectrum is discontinuous. In 1885, Balmer proposed an empirical formula that allowed for the determination of the frequencies of these bands, known as hydrogen spectral lines. The relationship is as follows:

$$\lambda = b\frac{n^2}{n^2-4}$$

Where:

b is a constant specific to each element.

n is an integer (3, 4, 5, ...) with $n \leq 3$

Since $\nu = \frac{c}{\lambda} \rightarrow \lambda = \frac{c}{\nu}$,

we substitute in the previous relationship:

$$\frac{c}{\nu} = b\frac{n^2}{n^2-4} \quad \Rightarrow \quad \frac{\nu}{c} = \frac{(n^2-4)}{bn^2}$$

$$\Rightarrow \nu = \frac{c}{bn^2}(n^2-4)$$

So if we put $b = \frac{4}{R_H}$:

$$\Rightarrow \nu = \frac{cR_H}{4n^2}(n^2-4)$$

$$\Rightarrow \frac{\nu}{c} = R_H\left(\frac{1}{4} - \frac{1}{n^2}\right)$$

$$\boxed{\Rightarrow \bar{\nu} = \frac{\nu}{c} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)}$$

This is the relationship proposed by Balmer:

$\bar{\nu}$= wavenumber, unit (cm^{-1}).

R_H= Rydberg constant: (109677.7 cm^{-1})

n = an integer

n = 3: red

n = 4: blue

n = 5: cyan

n = 6: violet

ν = frequency of light (Hz).

c = speed of light: C = 3 × 10^8 m/s

These are called the Balmer series of lines emitted by hydrogen, and Bohr provided the theoretical explanation for this empirical relationship.

Bohr's Postulates:

Bohr applied quantum theory to the electron of the hydrogen atom and theoretically calculated the frequencies of the lines in the hydrogen spectrum.

To find this, he proposed the following postulates:

1- Electrons revolve around the nucleus in circular orbits.

2- The energy of an electron moving around the nucleus cannot take any value. Each orbit corresponds to a specific energy value, and electrons are only allowed to transition between orbits of certain semi-diameters. An electron that occupies an orbit is in a stable energy state, and its energy does not change as long as it remains in that orbit.

3- When an electron in an allowed stable orbit (E2) transitions to another allowed stable orbit (E1) with lower energy (E1 < E2), it loses energy in the form of electromagnetic radiation with a frequency ν, represented by the relationship:

$$\Delta E = E_2 - E_1 = h\nu \text{ -4}$$

Where h = Planck's constant.

ν = frequency of the emitted radiation.

Here, ΔE= $E_2 - E_1$ is the energy lost by the electron.

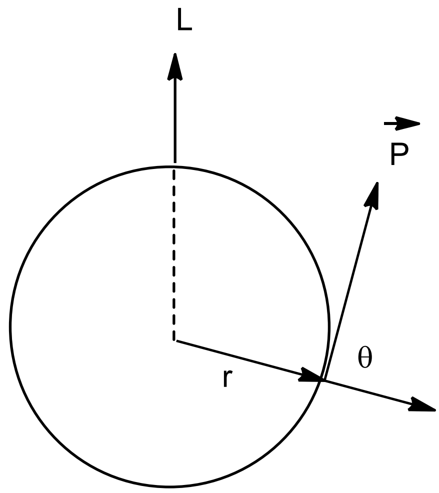
4-The angular momentum mvr of the electron, which is the quantized angular momentum mv with respect to the center of the orbit with a radius r, is also quantized and can only change in integral multiples of h/2π:

$$mvr = n\frac{h}{2\pi}$$

$$\theta = \frac{\pi}{2}$$

$\vec{L} = \vec{r}$ 人

$$L = rp\sin\theta$$
$$L = rp$$
$$L = rmv$$

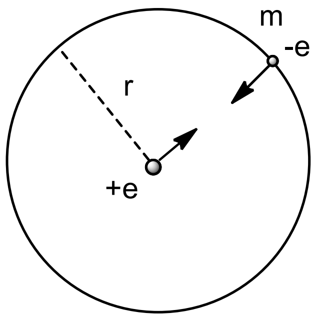


Bohr's postulates allowed him to theoretically derive a relationship similar to Balmer's empirical equation, which enables the calculation of the same experimental frequencies found by Balmer.

Calculating the Frequencies of Hydrogen Emissions:

A- Calculating Bohr's Semi-Diameters:

Here, we will provide a simplified calculation that allows us to establish a relationship similar to Balmer's.



- Consider a hydrogen atom nucleus with a charge of (+e).
- An electron with a charge (-e) orbits around it in an orbit with a semi-diameter r around the nucleus.
- The electrostatic force that attracts the electron toward the nucleus is described by Coulomb's law with the equation:

$$F = \frac{Kq.q\prime}{r^2} \Rightarrow F = -K\frac{e^2}{r^2} \rightarrow (1) \quad -$$

F < 0: because it's an attractive force.

F > 0: when it's repulsive.

In terms of units used, K is defined as follows:

K = 1 in the CGS system.

K = 9 × 10^9 in the MKSA system.

To prevent the electron from falling onto the nucleus, the attractive force of the electron toward the nucleus must equal the central repulsive force F', which is given by the following equation:

$$F' = m\frac{v^2}{r} \rightarrow (2)$$

By making (1)=(2):

$$\Rightarrow F' = |F|$$

$$\Rightarrow \frac{mv^2}{r} = \frac{ke^2}{r^2}$$

$$\Rightarrow r = K\frac{e^2}{mv^2} \rightarrow (3)$$

According to Bohr's fourth axiom, which says that the length of the angular momentum of the electron is quantized.

$$mvr = n\frac{h}{2\pi} \Rightarrow v = n\frac{h}{2\pi mr}$$

By substituting in Equation 3 the value of v:

$$r = K\frac{e^2}{m(\frac{n^2h^2}{4\pi^2m^2r^2})} = K\frac{e^2 \times 4\pi^2m^2 \times r^2}{n^2h^2}$$

$$\Rightarrow \quad r = \frac{n^2h^2}{k \times 4\pi^2 e^2 m}$$

$$\Rightarrow \quad r = a_0 = \frac{h^2}{4\pi^2 kme^2} \qquad\qquad n = 1$$

$$\Rightarrow \quad a_0 = 0.529A°$$

Where: m = electron mass: m=9.1×10^{-28} g or m=9.1×10^{-31} kg

v = electron speed.

r = radius of the orbit.

e = electron charge: 1.6×10^{-19} (C)

Regarding the potential energy of the electron (Ep).

Just like any object near the Earth's surface has gravitational potential energy because it's attracted by the Earth, the electron near the nucleus also possesses potential energy due to the attractive force exerted by the nucleus. This potential energy becomes greater the farther the electron is from the nucleus.

Let's provide the calculation for the potential energy of an electron located at a distance r from the nucleus.

By definition: the potential energy (Ep) of a charge q' at a distance r from a charge q is the work done by the external surroundings to move the charge q' from infinity to a distance r from the charge q, under the condition that the energy of the charge becomes zero at an infinite distance.

This means that the potential energy Ep is equal to the work of the Coulomb force when the charge moves from r to infinity. For this reason, it's also known as the Coulombic energy.

$$\mathrm{E_p} = \int_r^\infty dw = \int_r^\infty K\frac{qq'}{r^2}dr$$

$$= Kqq'\int_r^\infty \frac{dr}{r^2}$$

$$\mathrm{E_p} = Kqq'\int_r^\infty r^{-2}\,dr = Kqq'\left[-\frac{1}{r}\right]_r^\infty$$

$$= Kqq'\left[-\frac{1}{\infty} - (-\frac{1}{r})\right]$$

$$= Kqq'\left[0 + \frac{1}{r}\right]$$

$$\boxed{\mathrm{E_p} = \frac{Kqq'}{r}}$$

In the case of the hydrogen atom: q'=(-e), q=(+e):

$$\Rightarrow \mathrm{E_p} = \mathrm{K}\frac{(+e) \times (-e)}{r}$$

$$\boxed{\Rightarrow \mathrm{E_p} = -\mathrm{K}\frac{e^2}{r}}$$

From the previous equation No. (3):

$$v^2 = k\frac{e^2}{r.m}$$

h = Planck's constant such that:

$$\mathrm{h} = 6.61 \times 10^{-27}(erg \times s) = 6.61 \times 10^{-34}(j.s)$$

n = integer.

K= Coulomb's law constant: $K = 9 \times 10^9(c^{-2}.N.m^2)$

$$\mathrm{r} = \frac{1 \times (6\,.\,6 \times 10^{-34}(j.s))^2}{4 \times (3{,}14)^2 \times 9 \times 10^9 \times (1{,}6 \times 10^{-19}c)^2 \times 9{,}1 \times 10^{-31}(kg)}$$

$$r = \frac{43{,}6921 \times 10^{-68}(j^2.s^2)}{4 \times 9{,}8696 \times 9 \times 10^9 \times 2{,}56 \times 10^{-38}c^2}$$

$$r = \frac{43{,}6921 \times 10^{-68}}{909{,}582 \times 10^{-29} \times 9{,}1 \times 10^{-31}}$$

$$r = \frac{43{,}6921 \times 10^{-68}}{909{,}582 \times 10^{-29} \times 9{,}1 \times 10^{-31}}$$

$$r = \frac{43{,}692 \times 10^{-68}}{8277{,}1962 \times 10^{-60}}$$

$$\mathrm{r} = 5\,{,}2786 \times 10^{-11}\,m$$

$$\mathrm{r} = 0\,{,}528 \times 10^{-10} m$$

$$\Rightarrow \mathrm{r} = 0{,}528 \times A^{\circ}$$

$$\Rightarrow a_0 = 0\,{,}53\mathrm{A}^{\circ}$$

As we saw, this relationship allowed us to calculate the radius of the Bohr orbit when n = 1:

B- Calculating the energy of stable states:

The total energy E_T of an electron in its orbit (stable state) is equal to the sum of its kinetic energy + its potential energy.

$$E_T = \mathrm{E_p} + \mathrm{E_c}$$

$$E_{\mathrm{T}} = -k\frac{e^2}{r} + \frac{1}{2}mv^2$$

By substituting v^2 from (3):

$$E_{\mathrm{T}} = -K\frac{e^2}{r} + \frac{1}{2}\, m\, K\frac{e^2}{rm}$$

$$= -K\frac{e^2}{r} + \frac{ke^2}{2r}$$

$$= Ke^2\left[-\frac{1}{r} + \frac{1}{2r}\right]$$

$$E_T = -\mathrm{K}\frac{e^2}{2r}$$

When r is replaced by its value calculated in the previous relationship:

$$r = \frac{n^2h^2}{4\pi^2kme^2}$$

$$E_T = -K\frac{e^2 \times 4\pi^2 Kme^2}{2n^2h^2}$$

$$E_T = -\frac{4\pi^2 K^2 me^4}{2n^2h^2}$$

$$E_T = -\frac{2\pi^2 K^2 me^4}{n^2h^2}$$

Where:

E_T = Total energy of the electron in erg or J (joules).

m = Mass of the electron in g or kg.

e = Charge of the electron in units of UES CGS or in C (coulombs).

h = Planck's constant in erg·s or J·s.

n = Natural number.

K = Coulomb's constant = 1 or $\frac{9\times10^9}{(MKSA}$ (MKSA).

$$E_T = -\frac{2\pi^2 K^2 me^4}{h^2}\frac{1}{n^2}$$

Notes:

- We observe that as n ↗ increases, the total energy of the electron E_T ↗ increases, and its absolute value decreases ↙ | E_T |. Also, orbits with larger n values are more energetic.
- If we recall Bohr's fourth postulate: quantization of angular momentum

$$mvr = \frac{nh}{2\pi}$$

the largest radius relates to the largest value of n, and this is agreed upon in the total Bohr energy.

- The farther the electron is from the nucleus, the more energy it has.
- The electron closest to the nucleus is in a more stable state (lower energy).
- This explains why an electron in an excited state tends to quickly return to its ground state by emitting its energy in the form of quantized radiation hν.

C - Calculating the Frequency of Radiation:

When an electron jumps from an orbit with principal quantum number n_2 to an orbit with principal quantum number n_1 (where $n_1 < n_2$), it loses energy and emits radiation of energy hν. This radiation frequency ν can be easily calculated.

$$\mathrm{E}_2 - \mathrm{E}_1 = h\nu$$

$$\frac{-2\pi^2K^2me^4}{h^2n_2^2} + \frac{2K^2me^2}{h^2n_1^2} = h\nu$$

$$\frac{-2\pi^2K^2me^4}{h^2}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) = h\nu$$

$$\frac{2\pi^2K^2me^4}{h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = h\nu$$

$$\nu = \frac{2\pi^2k^2me^4}{h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\nu = \frac{C}{\lambda} = \bar{\nu}C = \frac{2\pi^2k^2me^4}{h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\boxed{\bar{\nu} = \frac{2\pi^2k^2me^4}{h^3c}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}$$

This is Balmer's experimental relationship:

$$\bar{\nu} = R_{\mathrm{H}}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$n_1=2$:

$$\bar{\nu} = R_{\mathrm{H}}\left(\frac{1}{2^2} - \frac{1}{n_2^2}\right)$$

Rydberg constant =

$$\boxed{R_H = \frac{2\pi^2k^4me^4}{h^3C}}$$

We find that the value of R_H = 109500cm^{-1}

This value is in good agreement with Balmer's experimental R_H value, which is equal to 109677 cm^{-1}

Series of Hydrogen Spectra

The success of Bohr's atomic model doesn't end with explaining the visible spectra of hydrogen. It's important to note that the Balmer formula represents a special case of Bohr's formula where n_1=2 (which is constrained to this value), while in Bohr's model, the quantum number n is required to take integer values, except for zero, where the orbit has a zero radius (r=0).

Furthermore, the discovery of new radiations emitted by hydrogen was a result of Bohr's model. These radiations are related to different n_1 values other than 2 (i.e., $n \neq 2$) and were not previously observed because they lie in the infrared (IR) and ultraviolet (UV) regions.

a) Lyman Series:

This series of lines was discovered by Lyman in the far ultraviolet region. It contains lines with high frequencies compared to the visible spectrum. It is related to n_1=1, and n2 can take values such as n_2=2, 3, 4, and so on.

b) Paschen Series:

Discovered by Paschen, this series is observed in the infrared region. It is related to n_1=3, and n_2 can take values such as n_2=4, 5, 6, and so on.

c) Brackett Series:

Brackett discovered this series, which is also observed in the infrared region. It is related to n_1=4, and n_2 can take values such as n2=5, 6, 7, and so on.

d) Pfund Series:

Pfund observed this series in the infrared region as well. It is related to n_1=5, and n2 can take values such as n_2=5, 6, 7, and so on.

It's worth noting that as we move away from the nucleus, the energies of the orbits gradually become closer to each other. The change in the quantum number n (by one) doesn't significantly affect the value of the orbit's radius r, as per Bohr's fourth postulate $mvr = n\frac{h}{2\pi}$, especially when the orbit's radius is large.

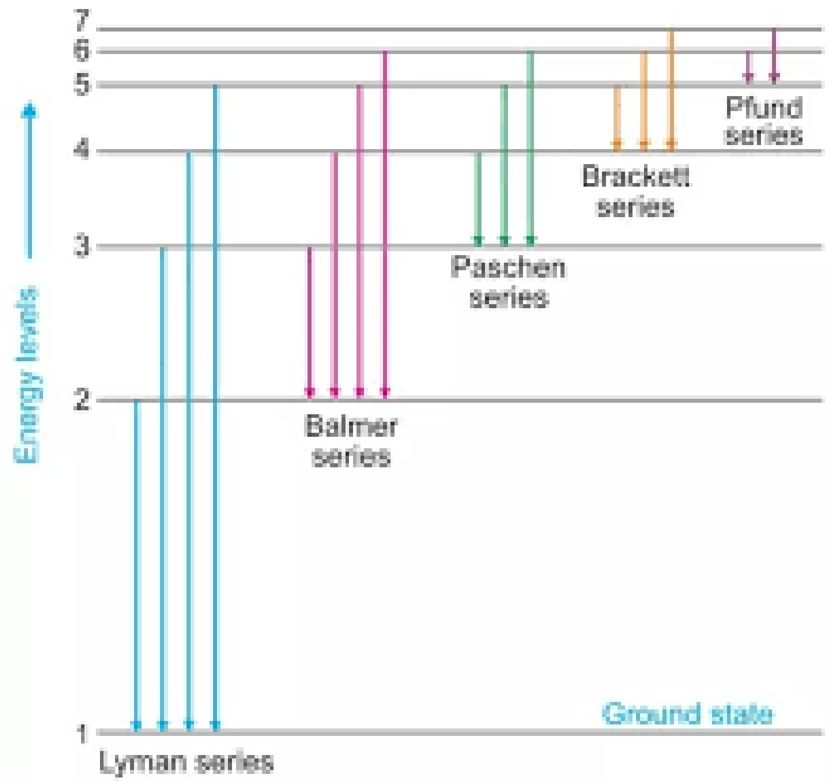
Notes:

1- Orbits are sometimes denoted with Latin letters: n=1 as K.

- n=2 as L.
- n=3 as M.
- n=4 as N.

2- The lines labeled Kα , Kβ, and Kγ are associated with transitions.

$$from\ \mathrm{n} = 3 \overset{to}{\rightarrow} \mathrm{n} = 2$$
$$from\ \mathrm{n} = 4 \overset{to}{\rightarrow} \mathrm{n} = 2$$
$$from\ \mathrm{n} = 5 \overset{to}{\rightarrow} \mathrm{n} = 2$$



Application of Bohr's theory to hydrogen atoms:

We call atoms similar to hydrogen: ions that contain a single electron orbiting the nucleus, as in hydrogen, and which have an atomic number Z higher than one (1). As an example of this:

$$He^{+}, \qquad Li^{++}, Be^{+++}, \qquad B^{++++}$$

Since it contains one electron, we use the same conclusion as for hydrogen. The electrostatic force (F) between the nucleus with charge (+Ze) and the electron with charge (-e), which is:

$$F = -K\frac{Ze^2}{r^2}$$

The potential energy of the electron becomes:

$$E_p = -K\frac{Ze^2}{r}$$

Which makes the total energy take the value:

$$E_T = E_p + E_c$$

$$E_T = -\frac{2\pi^2K^2me^4}{h^2} \times \frac{Z^2}{n^2}$$

As for the wave number, it is:

$$\times Z^2\left(\frac{1}{\mathrm{n}_1^2} - \frac{1}{\mathrm{n}_2^2}\right)\overline{\nu} = \frac{2\pi^2K^2me^4}{h^3c}$$

And R_H is placed: $$R_H = \frac{2\pi^2k^2me^4}{h^3c}$$

$$\overline{\nu} = R_HZ^2\left(\frac{1}{\mathrm{n}_1^2} - \frac{1}{\mathrm{n}_2^2}\right)$$

When Z=2 in the case of He^+, then:

$$\overline{\nu} = 4R_H\left(\frac{1}{\mathrm{n}_1^2} - \frac{1}{\mathrm{n}_2^2}\right)$$

The theoretical values of the calculated wave numbers are in complete agreement with the experimental results, and thus Bohr's theory has achieved its goal.

- It have interpreted all the lines that hydrogen emits.
- It also explained the lines emitted by hydrogenoid.
- It also gives a physical meaning to the light radiation in the electric discharge tube, regardless of the type of gas.

Multiple-electron atoms (Screening Effect):

The state of multiple-electron atoms is more complex. In this case, we also observe emission spectra related to electronic jumps or transitions.

If we consider an electron jumping from an orbit with quantum number (i) to an orbit with a lower quantum number (j):

There exists a screen between the electron and the nucleus, formed by inner electrons.

This screen is characterized by a constant called the shielding constant, denoted by σ. As a result, the force of attraction between the nucleus and the electron is equalized.

$$F = -K\frac{(Z-\sigma)e^2}{r^2}$$

So the total energy of this electron is:

$$E_n = -\frac{2\pi^2K^2me^4}{h^2} \times \frac{(Z-\sigma)^2}{n^2}$$

It results that:

$$\mathrm{E}_2 - \mathrm{E}_1 = h\nu$$

$$\Rightarrow \frac{2\pi^2K^2me^4(Z-\sigma)^2}{h^2}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) = h\nu$$

$$\nu = \frac{C}{\lambda} = \bar{\nu}C = \frac{2\pi^2k^2me^4(Z-\sigma)^2}{h^3}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

As for the wave number, it is:

$$\overline{\nu} = \frac{2\pi^2K^2me^4}{h^3c} \times (Z-\sigma)^2\left(\frac{1}{\mathrm{n}_1^2}-\frac{1}{\mathrm{n}_2^2}\right)$$

$$\Rightarrow \overline{\nu} = R_H(Z-\sigma)^2\left(\frac{1}{\mathrm{n}_1^2}-\frac{1}{\mathrm{n}_2^2}\right)$$

This is Balmer's formula that has been generalized to all elements. Mosely was able to calculate the constants (σ) in some cases of electronic transitions when he studied the emission spectra of elements using X-rays. Each of the lines $\mathrm{L}_\gamma, L_\beta, L_\alpha \ldots . \mathrm{K}_\gamma, K_\beta, K_\alpha$ is a linear function of the atomic number.

For a given line, $\sqrt{\nu} = K(Z-\sigma)$

From this relationship, the screening constant can be calculated, where K = the slope of the straight line in the relationship.